TRANSIENT, LAMINAR, FREE-FORCED CONVECTION WITH HEAT AND MASS TRANSFER FROM A VERTICAL ISOTHERMAL PLATE

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CERTIFICATE

CERTIFIED that the thesis titled, "TRANSIENT, LAMINAR, FREE-FORCED CONVECTION WITH HEAT AND MASS TRANSFER FROM A VERTICAL ISOTHERMAL PLATE" has been submitted by Ulhas R. Jagdhane under my supervision and that this work has not been submitted elsewhere for award of a degree.

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I.I.T. Kanpur
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Ulhas R. Jagdhane

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NOMENCLATURE

c	concentration
C	dimensionless concentration, $(c-c_\infty)/(c_w-c_\infty)$
C_p	specific heat of the fluid at constant pres
c'''	rate of species generation per unit volume
D	chemical molecular diffusivity
e	specific internal energy of the mixture
g	acceleration due to gravity
Gr	thermal Grashof number, $\beta g L^3(T_w-T_\infty)/\nu^2$
Gr^*	mass Grashof number, $\beta^* g L^3(c_w-c_\infty)/\nu^2$
h	local heat-transfer coefficient, $-K(\partial T/\partial Y)_w/$
h_D	local mass-transfer coefficient, $-D(\partial c/\partial Y)_w/$
$\bar{h}$	average heat transfer coefficient, $\frac{1}{L}\int_0^L h\,dx$
$\bar{h}_D$	average mass-transfer coefficient, $\frac{1}{L}\int_0^L h_D\,dx$
i	represents time
j	represents location in X-direction
k	represents location in Y-direction
K	thermal conductivity of the fluid
L	length of the flat plate
$\dot{m}$	vertical mass flux rate
N	buoyancy ratio parameter, $\beta^*(c_w-c_\infty)/\beta(T_w-T_\infty)$
Nu_x	instantaneous local Nusselt number, $\frac{hx}{K}$
Nu_m	instantaneous mean Nusselt number, $\frac{\bar{h}L}{K}\,Gr^{-1/}$
p	pressure
Pr	Prandtl number, ν/α

$\dot{q}$	heat flux rate
q'''	rate of energy generation per unit volume
Sc	Schmidt number, ν/D
Sh_x	instantaneous local Sherwood number, $\frac{h_D x}{D}$
Sh_m	instantaneous mean Sherwood number, $\frac{\bar{h}_D L}{D} Gr^{-1/4}$
t	time
T	temperature
u	x-velocity component
U	dimensionless X-velocity component, $\frac{uL}{\nu}\ Gr^{-1/2}$
v	y-velocity component
V	dimensionless Y-velocity component, $\frac{vL}{\nu}\ Gr^{-1/4}$
$\vec{V}$	velocity vector
x	spatial coordinate along the plate
X	dimensionaless spatial coordinate along the plate,
y	spatial coordinate normal to the plate
Y	dimensionless spatial coordinate normal to the plate, $\frac{y}{L}\ Gr^{1/4}$
α	thermal diffusivity, $K/\rho C_p$
$\alpha_k, \alpha'_k, \alpha''_k$	lower diagonal elements in tridiagonal matrices for momentum, energy, and species equations respectively
β	volumetric coefficient of thermal expansion
β^*	volumetric coefficient of expansion with concentration
$\beta_k, \beta'_k, \beta''_k$	main diagonal elements in tridiagonal matrices for momentum, energy, and species equations respectively
δ	velocity boundary layer thickness

δ_c	concentration boundary layer thickness
δ_t	thermal boundary layer thickness
$\Delta\tau$	dimensionless time-step
ΔX	dimensionless step size in X-direction
ΔY	dimensionless step-sixe in Y-direction
θ	dimensionless temperature, $(T-T_\infty)/(T_w-T_\infty)$
λ	relaxation factor
μ	dynamic viscosity of the fluid
ν	kinematic viscosity of the fluid, μ/ρ
τ	dimensionless time, $\frac{t\nu}{L^2}\,Gr^{1/2}$
ρ	density of the fluid
φ	function associated with the dissipation of energy
Φ_R	ratio of step sizes
$\varphi_k, \varphi_k', \varphi_k''$	elements of known right hand side column vectors of momentum, energy and species equations respectively
$\Omega_k, \Omega_k', \Omega_k''$	upper diagonal elements in tridiagonal matrices for momentum, energy, and species equation respectively
ξ	notation used for $-(\frac{\partial\theta}{\partial Y})\Big\vert_{Y=0}$
ζ	notation used for $-(\frac{\partial C}{\partial Y})\Big\vert_{Y=0}$

Subscripts

C	based on species concentration
U	based on velocity
w	at the surface of the plate
x	based on the distance from the leading edge of the plate
θ	based on temperature
∞	free stream conditions

Superscript

1	iteration number

Chapter 1

INTRODUCTION

There are many transport processes which occur in nature and in man-made devices in which flow arises simply due to the gradientsof density, temperatures, and/or chemical composition in a body force field, such as the gravitational field. Ever since the pioneering efforts by Lorenz [1] in 1881, such processes have been of considerable interest to engineers and scientists because of their numerous applications.

Processes in which buoyancy as the driving force arises solely due to the temperature difference have received considerable attention for both steady and transient, internal and external, and laminar and turbulent flows with several additional conditions and effects such as combined free and forced convection, etc. However, buoyancy effects resulting from concentration gradients in multicomponent mixtures can be just as important in generating fluid motion as the temperature gradients, as pointed out by Gebhart and Pera [2]. Fields of interest in which combined heat and mass transfer, under the condition of free convection are frequently encountered are : the evaporation of water from the surface of a water body in the absence of strong winds, as from ponds and lakes; drying processes in nature; distribution of temperature and moisture over agricultural fields and groves

of fruit trees; damage of crops due to freezing; formation and dispersion of fog; pollution of the environment; technological applications such as design of chemical processing equipment, etc.

In a large number of important applications, however, the convective process is neither predominantly natural nor predominantly forced; both modes being significant. The question then is whether the forced convection masks the natural convection, and if so, under what conditions? The answer to this question is provided here.

Consideration of transient natural convection is also important in many technological applications, since the heat transfer rates vary considerably during the transient stage. For given energy inputs this may result in over-heating and in consequent damage to various components of the systems, furnaces, electronic systems etc, which have, therefore to be designed to withstand the transients during the start up and shut down operations. We therefore consider the unsteady natural and forced convection in the presence of temperature and concentration gradients over a vertical flat plate. The flat plate provides the simplest geometry so that effects other than geometrical can be isolated.

Earlier developments :

One of the earliest studies with combined heat and mass transfer known to us is that of Somers [3]. It is concerned

with the combined thermal and species diffusion driven flow that would arise adjacent to a wetted isothermal vertical surface in a non-saturated atmosphere. The condition of very small diffusing species concentration was used and an integral method analysis was carried out for uniform surface temperature and uniform diffusing species concentration. The principal results were a transport relation and the indication that a combined driving force might be written in which the species diffusion contribution is modified by the square root of the Lewis number, i.e, $\sqrt{Le}$. The analysis is expected to be reasonable around Prandtl and Schmidt numbers of 1.0, with one buoyancy effect being very small compared to the other. Mathers et al. [4] formulated the same problem in terms of the boundary layer differential equations resulting from momentum, energy, and chemical species conservations at low concentration. Neglecting inertia effects the resulting equations were solved on an analogue computer for $Pr = 1.0$ and $Sc = 0.5 - 10.0$ for ratio of species and thermal diffusion buoyancy effects of 1.0 and 0.5. The resultant transport information appears to support the $\sqrt{Le}$ factor of Somers.

Possibility of similarity solutions for combined buoyancy effect flows formulated within the limitations assumed by Gill et al. [5] were considered by Lowell and Adams [6]. The only similarity solution found was that on an isothermal vertical surface. Results of numerical analysis of above similarity formulation were presented for a subliming organic

surface in air. Numerical difficulties that arose when buoyancy effects were opposed were noted and termed as flow instabilities, without any satisfactory justification.

Den Bounter [7] reports an experimental study of simultaneous thermal and chemical species diffusion by an electrochemical method between a vertical copper plate maintained at constant temperature and a copper sulphate-sulphuric acid solution. Measurements were made with the two buoyancy effects aiding and opposing each other. The mass and heat transfer parameters, correlated in terms of a combined buoyancy effect, calculated with the $\sqrt{Le}$ term, agree well with a single curve for the effects aiding each other. However for the two buoyancy effects opposing each other the disagreement from the single curve is random and over 30 percent in magnitude.

Bottemanne [8] has also considered steady state simultaneous heat and mass transfer along a vertical flat plate. Solution to the boundary layer equations was obtained only for $Pr = 0.71$ and $Sc = 0.63$. His theoretical solution agrees well with his experiments on heat transfer with simultaneous water evaporation into air.

The problem of combined forced and natural convection has been treated by Lloyd and Sparrow [9], covering conditions ranging from pure forced convection flow to combined flows with strong natural convection contribution, for an isothermal vertical flat plate. But buoyancy force arising only due to a

temperature difference is considered. The method of similarity is employed and numerical results are presented for Pr values ranging from 0.003 to 100.

Some experimental work has also been carried out on mixed convection from a vertical surface but neglecting buoyancy effect due to concentration gradient. Kliegel [10] employed interferometric methods to determine the local heat transfer rates from a vertical surface located in an air stream. This data was found to be in very good agreement with the analytical results of Lloyd and Sparrow.

Gryzagoridis [11] considered the natural convection flow over an isothermal vertical surface with aiding external flow and determined the local velocity and temperature profiles and the heat transfer rates. Good correlation between theory and experiment was obtained for the temperature profiles. The correlation was found to be very good for the heat transfer results, as expected from the agreement of temperature profiles. The limits of forced and free convection regions were also determined for Pr = 0.72.

Numerical solution for boundary layer equations for a transient free convection (buoyancy effects due to temperature gradient alone) over a vertical surface,subjected to a step change in the surface temperature, have been obtained by Hellums and Churchill [12]. The results converge to steady state values at large time and show a minimum in the Nusselt number during

the transient stage, as found earlier by Siegel.

Gebhart and Pera [2] studied laminar natural convection flows resulting from combined buoyancy mechanisms over a vertical flat plate in terms of similarity solutions. Over a range of Schmidt numbers,both aiding and opposing buoyancy effects were considered for air and water and solutions were obtained. The results show many interesting effects on velocity, heat and mass transfer, and on laminar stability. In this study stratification is neglected and only power law variations, $t-t_\infty = N_t x^n$, $c-c_\infty = N_c\ x^n$ are considered in order to make the similarity solution work. Gebhart and Pera studied the problem for steady state. However, the added constraint, brought in because of the transient terms, does not allow problems of practical importance to be studied by the similarity variable method, its application being restricted to particular forms of the surface temperature distribution. A numerical integration method may be considered but the procedure is complex one and does not yield desired physical insight into the process.

More recently Callhan and Marner [13] have considered transient laminar free convection along a vertical, isothermal flat plate, arising from buoyancy forces created by both temperature and concentration gradients. The coupled nonlinear partial differential equations are solved numerically using an explicit finite difference scheme. Results were obtained for $Pr = 1.0$ and range of Schmidt numbers and for aiding mass

diffusion buoyancy forces ($N > 0$). Steady state local Nusselt and Sherwood numbers were compared with the results of Gebhart and Pera. Largest deviation was observed at $X = 0.10$ with a difference of 4.2 percent while excellent agreement was found at the trailing edge of the plate with a difference of only 0.75 percent.

Present Work :

The purpose of the present study is to investigate the problem of transient, laminar, combined forced and natural convection along an isothermal vertical plate which is subjected to a step-change in temperature and concentration. The study covers conditions ranging from pure natural convection to strong forced convection. The coupled nonlinear partial differential equations are solved numerically by a highly implicit finite difference procedure.

There are many interesting aspects of such flows, such as the resulting transport characteristics, the influence of combined buoyancy force effects and combined free and forced convection on the stability of the boundary layers, and the effects of the values of relative transport parameters, the Prandtl and Schmidt numbers. Of particular interest in this study are :

i) The effect of the buoyancy forces due to mass transfer on the transient velocity profiles, temperature profiles, Nusselt number and Sherwood number.

ii) The effect of free stream velocity U_∞ on the Nusselt and Sherwood numbers, and on the transient velocity, temperature and concentration profiles.

Chapter 2

ANALYSIS

2.1 Governing Equations :

Fundamental physical processes that occur in natural-convection flows are essentially the same as those occuring in any fluid flow and diffusion processes. Therefore the basic equations used to interpret and analyse natural-convection flows are the same. There is however one fundamental difference between natural convection and forced convection. In natural convection fluid motion arises mainly from buoyancy and not from imposed motion or pressure difference. The buoyancy force arises due to the action of body force, usually gravity, on the density differences in a body of fluid, which ~~results~~ from temperature (and/or species-concentration) differences, which in turn are governed by the type of diffusion processs present. The diffusion processes which may be occuring simultaneously are coupled together resulting in much greater complexity and difficulty in treating this problem.

The basic equations are continuity, Navier-Stokes, energy and mass diffusion resulting from various conservation laws. These equations, in general form are [14]

continuity :

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho \vec{V}) = 0 \qquad (2.1)$$

momentum :

$$\rho \frac{D\vec{V}}{Dt} = \rho\left[\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\vec{\nabla})\vec{V}\right]$$

$$= \rho \vec{g} - \vec{\nabla} p + \mu \nabla^2 \vec{V} + \frac{\mu}{3} \vec{\nabla}(\vec{\nabla}.\vec{V}) \tag{2.2}$$

energy :

$$\rho \frac{De}{Dt} = \rho\left[\frac{\partial e}{\partial t} + (\vec{V}.\vec{\nabla}) e\right]$$

$$= \vec{\nabla}.(K\vec{\nabla} T) + q''' - p\, \vec{\nabla}.\vec{V} + \mu\varphi \tag{2.3}$$

species :

$$\frac{\partial c}{\partial t} + (\vec{V}.\vec{\nabla})\, c = \vec{\nabla}.(D\vec{\nabla} c) + c''' \tag{2.4}$$

where $\vec{V}$ is the fluid velocity vector, T is the temperature, e is the specific internal energy of the mixture, c is the concentration of a single diffusing species defined as the ratio of mass of the species (in a given volume) to the mass of mixture in the same volume, q''' and c''' are the rates of energy and species generation respectively per unit volume, $\vec{g}$ is the gravitational force per unit volume, p is the pressure, μ, K, D are the molecular transport properties namely dynamic viscosity, conductivity, and mass diffusivity and φ is the function associated with the dissipation of energy.

2.2. Approximations :

Considering the fluid to behave as a perfect gas, we can rewrite the energy equation (2.3) as

$$\rho\, c_p \frac{DT}{Dt} = \vec{\nabla}.(K\vec{\nabla} T) + q''' + \frac{Dp}{Dt} + \mu\varphi \tag{2.5}$$

where C_p is the specific heat at constant pressure for the mixture.

The principal difficulties in the above equations (2.1), (2.2), (2.4) and (2.5) result mainly from the possible variation of transport properties μ , K and D on the one hand and density ρ on the other hand. Since μ , K and D are dependent primarily on temperature*, an appreciable variation occurs only in those processes involving large temperature differences. Hence these properties are assumed constant here. Their variation can however be easily accounted for in the numerical method.

The density differences are approximated, for processes not involving large temperature differences, by the Boussinesq approximation [15]. This simplification renders the continuity equation to the constant density form, and introduces into the momentum equation a buoyancy force arising from both temperature and concentration differences.

For natural convection flows, from the hydrostatic considerations, the pressure gradient $\vec{\nabla}P$ in the remote ambient fluid is $\rho_\infty \vec{g}$ where ρ_∞ is the density of ambient fluid.

$$\therefore \quad \rho \vec{g} - \vec{\nabla}P = \vec{g}\,(\rho - \rho_\infty) .$$

Now since the surface is vertical and co-ordinate x is assumed positive upwards, as shown in fig. 2.1 the only term of the body force is $-\rho g_x$ and $\frac{\partial P}{\partial x} = -\rho_\infty g_x$

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* variation with concentration neglected due to low mass fraction of diffusing species.

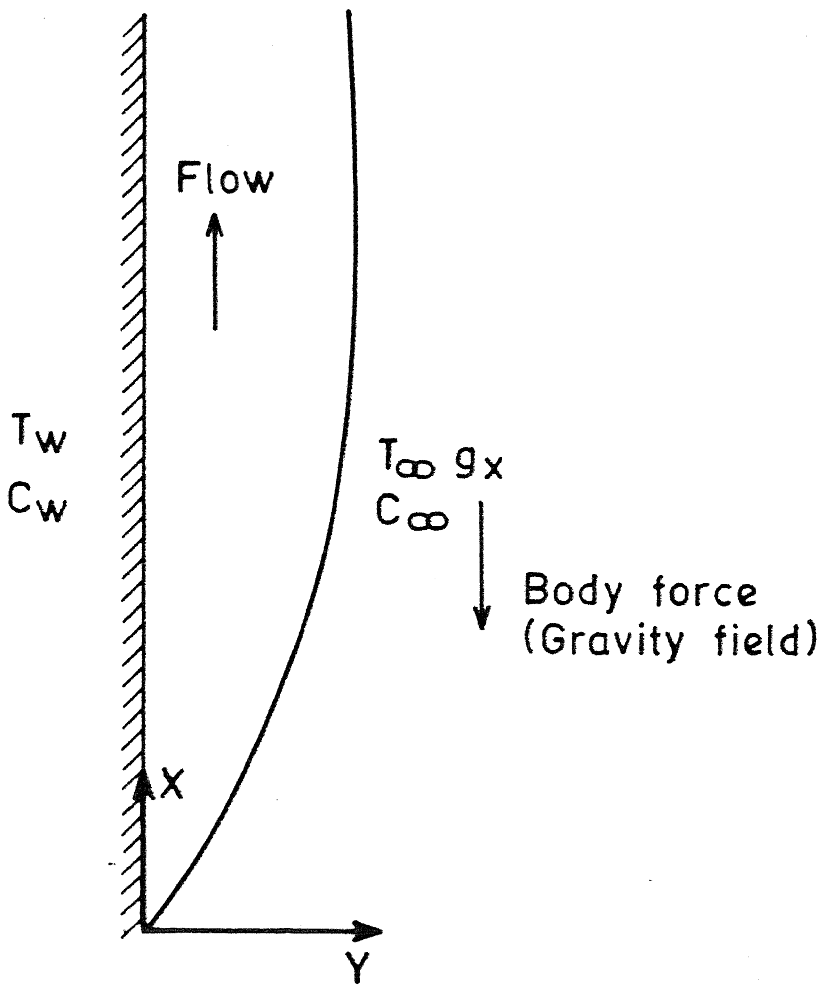

Fig. 2.1 Coordinate system for combined force and forced convection flow over a vertical flat plate.

$$\therefore \quad \rho\vec{g} - \vec{\nabla}P = g_x\,(\rho_\infty - \rho) \tag{2.6a}$$

The series expansion of $(\rho_\infty - \rho)$ in terms of T, p and c, at a given location can be written as

$$(\rho_\infty - \rho) = \rho\beta\,(T-T_\infty) + \rho\beta^*(c-c_\infty) \tag{2.6b}$$

where β is the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of expansion with concentration, T_∞ and c_∞ are the temperature and concentration respectively in the free stream.

The pressure term $\frac{DP}{Dt}$ in the energy equation (2.5) is negligible for gas flows of small vertical extent as is the case here. The viscous dissipation term is also negligible for small velocity flows. Moreover the rate of energy generation q''' and the rate of species generation c''' due to chemical reaction are assumed to be zero.

2.3. <u>Governing Equations for Natural Convection :</u>

With all approximations and assumptions discussed above the equations become

$$\vec{\nabla}.\vec{V} = 0 \tag{2.7}$$

$$\rho\,[\frac{\partial\vec{V}}{\partial t} + (\vec{V}.\vec{\nabla})\vec{V}] = \rho\,g_x\beta(T-T_\infty) + \rho\,g_x\beta^*\,(c-c_\infty) + \mu\nabla^2\vec{V} \tag{2.8}$$

$$\rho\,c_p\,[\frac{\partial T}{\partial t} + (\vec{V}.\vec{\nabla})T] = K\,\nabla^2\,T \tag{2.9}$$

$$\frac{\partial c}{\partial t} + (\vec{V}.\vec{\nabla})c = D\,\nabla^2\,c \tag{2.10}$$

The present study is concerned with the simple case of two dimensional flow where $\vec{V} = (u,v)$. In terms of the co-ordinate system shown in fig. 2.1, the equations (2.7) to (2.10) can be written as,

continuity :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.11}$$

x-momentum :

$$\rho\left(\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y}\right) = \rho\, g\beta(T-T_\infty) + \rho\, g\beta^*(c-c_\infty) + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2.12a}$$

y-momentum :

$$\rho\left(\frac{\partial v}{\partial t} + u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y}\right) = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{2.12b}$$

energy :

$$\rho\, c_p\left(\frac{\partial T}{\partial t} + u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y}\right) = K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{2.13}$$

species :

$$\frac{\partial c}{\partial t} + u\,\frac{\partial c}{\partial x} + v\,\frac{\partial c}{\partial y} = D\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right) \tag{2.14}$$

These equations can be further simplified by carrying out an order of magnitude analysis [15]. Let us first non-dimensionalize the variables as

$$\bar{u} = \frac{u}{U}\,,\quad \bar{v} = \frac{v}{U}\,,\quad \bar{x} = \frac{x}{L}\,,\quad \bar{y} = \frac{y}{L}\,,\quad \theta = \frac{T-T_\infty}{T_w-T_\infty}\,,\quad C = \frac{c-c_\infty}{c_w-c_\infty}$$

where L, U are the characteristic length and velocity, T_w and c_w are the wall temperature and concentration. Equations (2.11) to (2.14) in dimensionless form are

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{2.15}$$

$o(1) \quad o(1)$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \theta + NC + \frac{1}{\sqrt{Gr}}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) \tag{2.16a}$$

$o(1) \quad o(1)o(1) \quad o(\delta)o(\frac{1}{\delta}) \quad o(1) \quad o(1) \qquad o(1) \quad o(\frac{1}{\delta^2})$

$$\frac{\partial \bar{v}}{\partial t} + \bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = \frac{1}{\sqrt{Gr}}\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) \tag{2.16b}$$

$o(\delta) \quad o(1)o(\delta) \quad o(\delta)o(1) \qquad o(\delta) \quad o(\frac{1}{\delta})$

$$\frac{\partial \theta}{\partial t} + \bar{u}\frac{\partial \theta}{\partial \bar{x}} + \bar{v}\frac{\partial \theta}{\partial \bar{y}} = \frac{1}{Pr\sqrt{Gr}}\left(\frac{\partial^2 \theta}{\partial \bar{x}^2} + \frac{\partial^2 \theta}{\partial \bar{y}^2}\right) \tag{2.17}$$

$o(1) \quad o(1)o(1) \quad o(\delta)o(\frac{1}{\delta_t}) \qquad o(1) \quad o(\frac{1}{\delta_t^2})$

$$\frac{\partial C}{\partial t} + \bar{u}\frac{\partial C}{\partial \bar{x}} + \bar{v}\frac{\partial C}{\partial \bar{y}} = \frac{1}{Sc\sqrt{Gr}}\left(\frac{\partial^2 C}{\partial \bar{x}^2} + \frac{\partial^2 C}{\partial \bar{y}^2}\right) \tag{2.18}$$

$o(1) \quad o(1)o(1) \quad o(\delta)o(\frac{1}{\delta_c}) \qquad o(1) \quad o(\frac{1}{\delta_c^2})$

where $N = [\beta^*(c_w - c_\infty)]/[\beta(T_w - T_\infty)]$ measures the relative importance of chemical and thermal effects in causing the density differences which create the natural convection effect. $Gr = g\beta L^3 (T_w - T_\infty)/\nu^2$ is the thermal Grashof number, $Gr^* = g\beta^* L^3 (c_w - c_\infty)/\nu^2$ is the mass Grashof number, $Pr = \frac{\mu C_p}{K}$ is the Prandtl number, and $Sc = \frac{D}{\alpha}$ is the Schmidt number, and where the order of magnitude of each term is written under it according to the following estimation.

In the continuity equation $\frac{\partial \bar{u}}{\partial \bar{x}}$ must be of order 1 since both $\bar{u}$ and $\bar{x}$ are of order 1. Therefore $\frac{\partial \bar{v}}{\partial \bar{y}}$ is also of order 1.

Since $\bar{y}$ is of order δ for the hydrodynamic boundary layer, $\bar{v}$ is also of order δ. Clearly δ is the hydrodynamic boundary layer thickness. In the x-direction momentum equation (2.16a), we can therefore neglect $\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} = O(1)$ as compared to $\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} = O(\frac{1}{\delta^2})$. Also, viscous forces must be of the same order as the inertia forces and as the buoyancy forces. Therefore $\delta = O(Gr^{-1/4})$. Considering the y-momentum equation (2.16b) it is observed that the inertia and viscous terms are of order δ or less. Hence as compared to the x-momentum equation the whole of y-momentum equation can be neglected.

For conduction and convection terms in the energy equation (2.17) to be comparable δ_t = thermal boundary layer thickness = $O\,(Pr^{-1/2}\,Gr^{-1/4})$. Similarly an estimate of the concentration boundary layer thickness δ_c is $\delta_c = O(Sc^{-1/2}Gr^{-1/4})$. Also the terms $\frac{\partial^2\theta}{\partial \bar{x}^2}$ and $\frac{\partial^2 c}{\partial \bar{x}^2}$ are negligible as compared to $\frac{\partial^2\theta}{\partial \bar{y}^2}$ and $\frac{\partial^2 c}{\partial \bar{y}^2}$ respectively, and can therefore be neglected.

This analysis of relative order of magnitude yields the following system of boundary layer-type equations governing the distributions of u,v,T and c for transient free convection over a vertical flat plate.

continuity :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.19}$$

momentum :

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \beta g(T-T_\infty)+\beta^{*}g(c-c_\infty) \tag{2.20}$$

energy :

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad (2.27)$$

species :

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2} \qquad (2.22)$$

where we have returned to dimensional variables u, v, T and c and where ν is the kinematic viscosity (μ/ρ) and α is the thermal diffusivity $(K/\rho C_p)$.

2.4 Boundary Conditions And Initial Conditions :

For an isothermal plate, the boundary and initial conditions are

$$\left.\begin{array}{lll} u(x,y,0) = 0; & T(x,y,0) = T_\infty; & c(x,y,0) = c_\infty \\ u(x,0,t) = 0; & T(x,0,t) = T_w; & c(x,0,t) = c_w \\ u(0,y,t) = u_\infty; & T(0,y,t) = T_\infty; & c(0,y,t) = c_\infty \\ u(x,\infty,t) = u_\infty; & T(x,\infty,t) = T_\infty; & c(x,\infty,t) = c_\infty \\ v(x,y,0) = 0; & & \\ v(x,0,t) = 0; & & \end{array}\right\} \qquad (2.23)$$

where u_∞ is the free stream velocity in the same direction as that induced by free convection i.e. aiding flow. When $u_\infty = 0$ the problem changes to that of pure natural convection.

2.5. Non-dimensionalisation :

By the following choice of dimensionless parameters suggested by the above order of magnitude analysis

$$X = \frac{x}{L} \quad ; \quad Y = \frac{y}{L\, Gr^{-1/4}} \quad ; \quad \tau = \frac{t\ \nu\, Gr^{1/2}}{L^2}$$

$$U = \frac{uL}{\nu\, Gr^{1/2}} \quad ; \quad V = \frac{v\, L}{\nu\, Gr^{1/4}}$$

$$\theta = \frac{T-T_\infty}{T_w-T_\infty} \quad ; \quad C = \frac{c-c_\infty}{c_w-c_\infty}$$

equations (2.19) through (2.22) and initial and boundary conditions are expressed in dimensionless form as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad (2.24)$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + \theta + NC \qquad (2.25)$$

$$\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} \qquad (2.26)$$

$$\frac{\partial C}{\partial \tau} + U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \qquad (2.27)$$

$$\left.\begin{array}{lll}
U(X,Y,0) = 0 \ ; & \theta(X,Y,0) = 0 \ ; & C(X,Y,0) = 0 \\
U(X,0,\tau) = 0 \ ; & \theta(X,0,\tau) = 1 \ ; & C(X,0,\tau) = 1 \\
U(0,Y,\tau) = U_\infty \ ; & \theta(0,Y,\tau) = 0 \ ; & C(0,Y,\tau) = 0 \\
U(X,\infty,\tau) = U_\infty \ ; & \theta(X,\infty,\tau) = 0 \ ; & C(X,\infty,\tau) = 0 \\
V(X,Y,0) = 0 & & \\
V(X,0,\tau) = 0 & &
\end{array}\right\} \qquad (2.28)$$

where $U_\infty = \dfrac{u_\infty\, L}{\nu\, Gr^{1/2}} = Re\ Gr^{-1/2}$.

Equations (2.24) through (2.27) and (2.28) show that dependent variables U, V, θ and C are functions of the dimensionless spatial coordinates X and Y, dimensionless time τ, and dimensionless parameters N, Pr, Sc and U_∞. Note that for forced flow, $\tau = (t\ u_\infty)/(LU$ which is a proper dimensionless time.

N measures the relative importance of chemical and thermal diffusion in causing the density differences, which create the natural convection effect. N is equal to zero when there is no species diffusion body force and becomes infinite for no thermal diffusion*. The momentum equation (2.25) indicates that N is positive or negative according as the mass diffusion forces aid or oppose those of thermal diffusion. $U_\infty = \text{Re Gr}^{-1/2}$ is the parameter which determines the extent of contribution of forced convection to natural convection. $U_\infty = 0$ is the case of pure natural convection while $U_\infty > 0$ is aiding flow and $U_\infty < 0$ is the opposing flow. Negative values of U_∞ can cause separation [16] which can not be handled by the boundary layer type equations (2.24) to (2.27). We therefore consider $U_\infty \geq 0$ only.

In the case of pure forced convection, the Prandtl number ($Pr = \nu/\alpha$) relates the relative thicknesses of the momentum and thermal boundary layers δ and δ_t respectively. Similarly the Schmidt number ($Sc = \nu/D$) in the pure forced convective mass transfer, relates the momentum and concentration boundary layer thicknesses for δ and δ_c.

However, in the case of free convection or combined free and forced convection, in the presence of mass diffusion contribution to the buoyancy force, the relationship amongst δ, δ_t and δ_c is extremely complex and depends upon Sc, Pr, U_∞ and the buoyancy ratio parameter N.

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* For $N \to \infty$, Gr should be replaced by Gr* for non-dimensionalizatio

2.6. Heat and Mass Transfer Analysis :

It is a common practice to express heat transfer and mass transfer characteristics in terms of the flux rate divided by the temperature or concentration difference, causing the heat and mass transfer respectively. This ratio defines the heat and mass-transfer coefficients h and h_D respectively, with the help of which we can define the instantaneous local Nusselt and Sherwood numbers as

$$Nu_x = \frac{hx}{K} = \frac{\dot{q}\ x}{K(T_w - T_\infty)} \tag{2.29a}$$

$$Sh_x = \frac{h_D x}{D} = \frac{\dot{m}\ x}{D(c_w - c_\infty)} \tag{2.29b}$$

where $\dot{q}$ and $\dot{m}$ are the heat and mass flux rates respectively. For uniform values of temperature and concentration differences $(T_w - T_\infty)$ and $(c_w - c_\infty)$, local values, h and h_D , and average values $\bar{h}$ and $\bar{h}_D$ are of interest.

These values are expressed in the dimensionless form to obtain the instantaneous local Nusselt and Sherwood numbers respectively as follows

$$Nu_x = -\left(\frac{\partial \theta}{\partial Y}\right)\bigg|_{Y=0} X\ Gr^{1/4} \tag{2.30a}$$

$$Sh_x = -\left(\frac{\partial C}{\partial Y}\right)\bigg|_{Y=0} X\ Gr^{1/4} \tag{2.30b}$$

Since $Gr^{1/4}$ is a constant, let us merge it into Nu_x and Sh_x so as to redefine the instantaneous local Nusselt and Sherwood numbers as

$$Nu_x = -X \left(\frac{\partial \theta}{\partial Y}\right)\Big|_{Y=0} \qquad (2.31a)$$

$$Sh_x = -X \left(\frac{\partial C}{\partial Y}\right)\Big|_{Y=0} \qquad (2.31b)$$

The instantaneous mean Nusselt number is defined as

$$Nu_m = \left(\frac{\bar{h}L}{K}\right) \quad Gr^{-1/4} \qquad (2.32)$$

where $\bar{h} = \frac{1}{L} \int_0^L h \, dx = \int_0^1 h \, dX$

This yields

$$Nu_m = \int_0^1 \left(- \frac{\partial \theta}{\partial Y}\right)\Big|_{Y=0} dX \qquad (2.33a)$$

Similarly the instantaneous mean Sherwood number can be found from

$$Sh_m = \int_0^1 -\left(\frac{\partial C}{\partial Y}\right)\Big|_{Y=0} dX \qquad (2.33b)$$

Chapter 3

FINITE DIFFERENCE FORMULATION

Solutions of the coupled continuity, momentum, energy and species equations (2.24) - (2.27) subjected to initial and boundary conditions (2.28), were obtained using a numerical marching procedure. Numerical marching procedures are methods in which the solution is obtained in a step-by-step manner, always moving downstream through the flow field, and forward in time. A number of finite difference forms for the representation of the above equations are possible. These are the implicit, Crank-Nicholson form etc.

The choice of finite difference representation depends on many factors, including the problem itself, and the size and speed of computation desired. Explicit difference representations are those in which the unknown quantities in the equation may be solved for one at a time, as each step in the marching direction is taken. While simple to work with, explicit methods are prone to instability and require impracticably small step sizes in order to ensure stability. Implicit representations require the solution of a set of simultaneous equations for the unknowns as each step is taken in the downstream direction or in time. They have no stability constraints and thus allow large steps to be taken without any problem.

However, almost all formulations for transient problems, appearing in the literature are explicit. Callhan and Marner [13] too, have solved the present problem using explicit representation. We use the implicit formulation nevertheless in order to take advantage of its universal stability so long as $U \geq 0$. This means in particular that there is no restriction on the size of steps $\Delta\tau$ or ΔX. In using the explicit formulation, such small values of $\Delta\tau$ and ΔX are required that computations can become extremely time consuming. Since the matrices encountered in the implicit formulation discussed here are tridiagonal, the computational time required for a complete set of calculations at each step is approximately the same as that required for the explicit method, but much larger values of $\Delta\tau$ and ΔX are permitted by the implicit method. Thus the implicit method seems to be superior to the explicit method.

3.1 Finite-Difference Equations :

A non-uniform finite difference grid (Fig. 3.1) is imposed on the flow field. Step sizes ΔX, ΔY and $\Delta\tau$ are taken respectively in the X and Y direction and in time. Subscripts $j+1$ and k indicate the location of the point under consideration in x and y directions respectively, while the subscript $(i+1)$ indicates the current time.

The difference form selected for equations (2.25), (2.26) and (2.27) is highly implicit in that not only are all

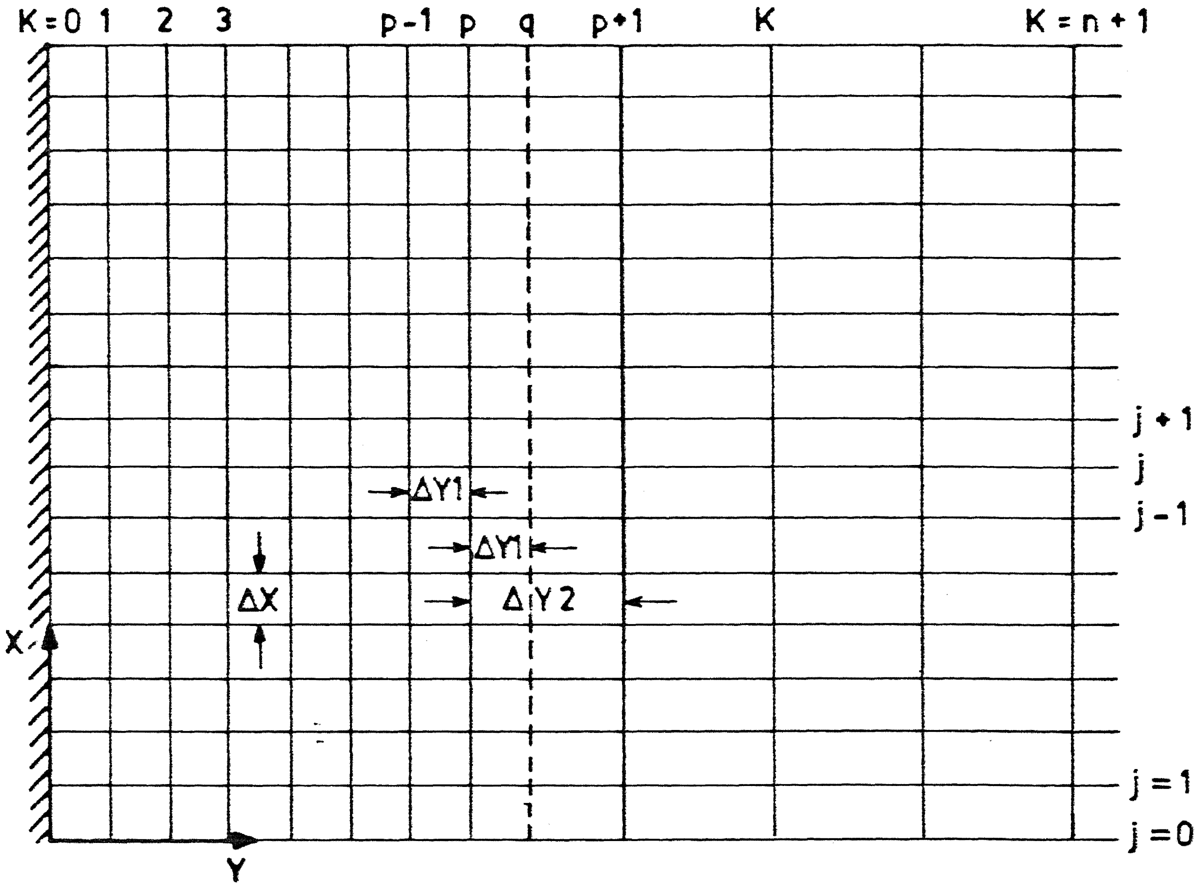

Fig. 3.1 Finite Difference grid with variable mesh size.

Y-derivatives evaluated at j+1 but the coefficients of non-linear convective terms are also evaluated at j+1. This is essential for free convection flow since if the usual implicit form is chosen it results in U velocity profile decreasing linearly from the plate to zero at whatever value of Y is chosen as infinity. This result is obviously incorrect. Hence the highly implicit scheme is used.

The finite difference forms chosen for equations (2.24) to (2.27) are

$$\frac{U_{j+1,k+1,i+1}-U_{j,k+1,i+1}}{\Delta X}+\frac{V_{j+1,k+1,i+1}-V_{j+1,k,i+1}}{\Delta Y}=0 \quad (3.1)$$

$$\frac{U_{j+1,k,i+1}-U_{j+1,k,i}}{\Delta\tau}+U_{j+1,k,i+1}\,\frac{U_{j+1,k,i+1}-U_{j,k,i+1}}{\Delta X}$$

$$+\,V_{j+1,k,i+1}\,\frac{U_{j+1,k+1,i+1}-U_{j+1,k-1,i+1}}{2(\Delta Y)}$$

$$=\frac{U_{j+1,k+1,i+1}-2U_{j+1,k,i+1}+U_{j+1,k-1,i+1}}{(\Delta Y)^2}+\theta_{j+1,k,i+1}$$

$$+\,N\,C_{j+1,k,i+1} \quad (3.2)$$

$$\frac{\theta_{j+1,k,i+1}-\theta_{j+1,k,i}}{\Delta\tau}+U_{j+1,k,i+1}\,\frac{\theta_{j+1,k,i+1}-\theta_{j,k,i+1}}{\Delta X}$$

$$+\,V_{j+1,k,i+1}\,\frac{\theta_{j+1,k+1,i+1}-\theta_{j+1,k-1,i+1}}{2(\Delta Y)}$$

$$= \frac{1}{Pr} \frac{\theta_{j+1,k+1,i+1} - 2\theta_{j+1,k,i+1} + \theta_{j+1,k-1,i+1}}{(\Delta Y)^2} \qquad (3.3)$$

$$\frac{C_{j+1,k,i+1} - C_{j+1,k,i}}{\Delta\tau} + U_{j+1,k,i+1} \frac{C_{j+1,k,i+1} - C_{j,k,i+1}}{\Delta X}$$

$$+ V_{j+1,k,i+1} \frac{C_{j+1,k+1,i+1} - C_{j+1,k-1,i+1}}{2(\Delta Y)}$$

$$= \frac{1}{Sc} \frac{C_{j+1,k+1,i+1} - 2C_{j+1,k,i+1} + C_{j+1,k-1,i+1}}{(\Delta Y)^2} \qquad (3.4)$$

Truncation error is of $o(\Delta X)$ and $o(\Delta Y)^2$ for momentum, energy and concentration equations, and of $o(\Delta X)$ and $o(\Delta Y)$ for the continuity equation. Since the differential formulation given above is non-linear, none of the techniques for linear algebraic equations may be employed. However, one very simple and effective iterative technique is used here.

To start with equations (3.1) to (3.4) are rewritten using superscripts to indicate on which iteration that value was obtained, for example $U^{(l)}_{j+1,k,i+1}$ is obtained on the $(l)^{th}$ iteration while $U^{(l+1)}_{j+1,k,i+1}$ is obtained on the $(l+1)^{th}$ iteration. In such a linearised form equations (3.1) to (3.4) are rewritten as

$$\frac{U^{(l+1)}_{j+1,k,i+1} - U_{j,k,i+1}}{\Delta X} + \frac{V^{(l+1)}_{j+1,k+1,i+1} - V^{(l+1)}_{j+1,k,i+1}}{\Delta Y} = 0 \qquad (3.5)$$

$$\frac{U^{(l+1)}_{j+1,k,i+1} - U_{j+1,k,i}}{\Delta\tau} + U^{(l)}_{j+1,k,i+1}\,\frac{U^{(l+1)}_{j+1,k,i+1} - U_{j,k,i+1}}{\Delta X}$$

$$+ V^{(l)}_{j+1,k,i+1}\,\frac{U^{(l+1)}_{j+1,k+1,i+1} - U^{(l+1)}_{j+1,k-1,i+1}}{2(\Delta Y)}$$

$$= \frac{U^{(l+1)}_{j+1,k+1,i+1} - 2U^{(l+1)}_{j+1,k,i+1} + U^{(l+1)}_{j+1,k-1,i+1}}{(\Delta Y)^2}$$

$$+ \theta^{(l)}_{j+1,k,i+1} + NC^{(l)}_{j+1,k,i+1} \qquad (3.6)$$

$$\frac{\theta^{(l+1)}_{j+1,k,i+1} - \theta_{j+1,k,i}}{\Delta\tau} + U^{(l+1)}_{j+1,k,i+1}\,\frac{\theta^{(l+1)}_{j+1,k,i+1} - \theta_{j,k,i+1}}{\Delta X}$$

$$+ V^{(l+1)}_{j+1,k,i+1}\,\frac{\theta^{(l+1)}_{j+1,k+1,i+1} - \theta^{(l+1)}_{j+1,k-1,i+1}}{2(\Delta Y)}$$

$$= \frac{1}{Pr}\,\frac{\theta^{(l+1)}_{j+1,k+1,i+1} - 2\theta^{(l+1)}_{j+1,k,i+1} + \theta^{(l+1)}_{j+1,k-1,i+1}}{(\Delta Y)^2} \qquad (3.7)$$

$$\frac{C^{(l+1)}_{j+1,k,i+1} - C_{j+1,k,i}}{\Delta\tau} + U^{(l+1)}_{j+1,k,i+1}\,\frac{C^{(l+1)}_{j+1,k,i+1} - C_{j,k,i+1}}{\Delta X}$$

$$+ V^{(l+1)}_{j+1,k,i+1}\,\frac{C^{(l+1)}_{j+1,k+1,i+1} - C^{(l+1)}_{j+1,k-1,i+1}}{2(\Delta Y)}$$

$$= \frac{1}{Sc}\,\frac{C^{(l+1)}_{j+1,k+1,i+1} - 2C^{(l+1)}_{j+1,k,i+1} + C^{(l+1)}_{j+1,k-1,i+1}}{(\Delta Y)^2} \qquad (3.8)$$

To continuity equation (3.5) and conservation equations (3.6) to (3.8) for momentum, energy and chemical species respectively are written in more useful form as

$$V^{(l+1)}_{j+1,k+1,i+1} = V^{(l+1)}_{j+1,k,i+1} - \frac{\Delta Y}{\Delta X}(U^{(l+1)}_{j+1,k+1,i+1} - U_{j,k+1,i+1}) \tag{3.9}$$

$$\left[\frac{-1}{(\Delta Y)^2} - \frac{V^{(l)}_{j+1,k,i+1}}{2(\Delta Y)}\right] U^{(l+1)}_{j+1,k-1,i+1} + \left[\frac{1}{\Delta\tau} + \frac{2}{(\Delta Y)^2} + \frac{U^{(l)}_{j+1,k,i+1}}{\Delta X}\right]$$

$$U^{(l+1)}_{j+1,k,i+1} + \left[\frac{-1}{(\Delta Y)^2} + \frac{V^{(l)}_{j+1,k,i+1}}{2(\Delta Y)}\right] U^{(l+1)}_{j+1,k+1,i+1}$$

$$= \frac{U_{j+1,k,i}}{\Delta\tau} + \frac{U^{(l)}_{j+1,k,i+1}\ U_{j,k,i+1}}{\Delta X} + \theta^{(l)}_{j+1,k,i+1}$$

$$+ N\ C^{(l)}_{j+1,k,i+1} \tag{3.10}$$

$$\left[\frac{-1}{Pr(\Delta Y)^2} - \frac{V^{(l+1)}_{j+1,k,i+1}}{2(\Delta Y)}\right] \theta^{(l+1)}_{j+1,k-1,i+1} + \left[\frac{1}{\Delta\tau} + \frac{2}{Pr(\Delta Y)^2} + \frac{U^{(l+1)}_{j+1,k,i+1}}{\Delta X}\right]$$

$$\theta^{(l+1)}_{j+1,k,i+1} + \left[\frac{-1}{Pr(\Delta Y)^2} + \frac{V^{(l+1)}_{j+1,k,i+1}}{2(\Delta Y)}\right] \theta^{(l+1)}_{j+1,k+1,i+1}$$

$$= \frac{\theta_{j+1,k,i}}{\Delta\tau} + \frac{U^{(l+1)}_{j+1,k,i+1}\ \theta_{j,k,i+1}}{\Delta X} \tag{3.11}$$

$$\left[\frac{-1}{Sc(\Delta Y)^2} - \frac{V^{(l+1)}_{j+1,k,i+1}}{2(\Delta Y)}\right] C^{(l+1)}_{j+1,k-1,i+1} + \left[\frac{1}{\Delta\tau} + \frac{2}{Sc(\Delta Y)^2}\right.$$

$$+ \frac{U^{(l+1)}_{j+1,k,i+1}}{\Delta X}] \; C^{(l+1)}_{j+1,k,i+1} + [\frac{-1}{Sc(\Delta Y)^2} + \frac{V^{(l+1)}_{j+1,k,i+1}}{2(\Delta Y)}] \; C^{(l+1)}_{j+1,k+1,i+1}$$

$$= \frac{C_{j+1,k,i}}{\Delta \tau} + \frac{U^{(l+1)}_{j+1,k,i+1} \; C_{j,k,i+1}}{\Delta X} \tag{3.12}$$

Equations (3.10) to (3.12) written for $k = 1(1)n$ constitute sets of n linear algebraic equations in n unknowns $U^{(l+1)}_{j+1,k,i+1}$, $\theta^{(l+1)}_{j+1,k,i+1}$ and $C^{(l+1)}_{j+1,k,i+1}$ respectively. Value of n is chosen large enough, so that on several points of the grid close to $k = n$ the U velocities are essentially that of free stream. Solution procedure and steps followed for the solution of the finite difference equations (3.9) to (3.12) to be explained later.

3.2 Heat and Mass Transfer Solution :

In finite difference form equations (2.31a) and (2.31b) for the instantaneous local Nusselt and Sherwood numbers can be written as

$$Nu_x = X \; \frac{3\theta_{j+1,0} - 4\theta_{j+1,1} + \theta_{j+1,2}}{2(\Delta Y)} \tag{3.13a}$$

$$Sh_x = X \; \frac{3C_{j+1,0} - 4C_{j+1,1} + C_{j+1,2}}{2(\Delta Y)} \tag{3.13b}$$

where three-point forward differences have been used to evaluate $(\frac{\partial \theta}{\partial Y})$ and $(\frac{\partial C}{\partial Y})$ at the plate. These differences involve an error of $o(\Delta Y)^2$.

For convenience in expressing the instantaneous mean Nusselt and Sherwood numbers, let ξ and ζ represent the following values at any instant

$$\xi = -\left(\frac{\partial\theta}{\partial Y}\right)\Big|_{Y=0} \quad \text{and} \quad \zeta = -\left(\frac{\partial C}{\partial Y}\right)\Big|_{Y=0} \tag{3.14}$$

Then the instantaneous mean Nusselt and Sherwood numbers can be written as (cf. equations (2.33a) and (2.33b))

$$Nu_m = \int_0^1 \xi \, dX \tag{3.15a}$$

$$Sh_m = \int_0^1 \zeta \, dX \tag{3.15b}$$

Using Simpson's rule [17], these can be written as

$$Nu_m = \frac{\Delta X}{3}\left[\xi_o+4\xi_1+2\xi_2+4\xi_3+\ldots+2\xi_{m-2}+4\xi_{m-1}+\xi_m\right] \tag{3.16a}$$

$$Sh_m = \frac{\Delta X}{3}\left[\zeta_o+4\zeta_1+2\zeta_2+4\zeta_3+\ldots+2\zeta_{m-2}+4\zeta_{m-1}+\zeta_m\right] \tag{3.16b}$$

where m must be an even number.

3.3. Computational Steps :

For the solution of finite-difference equations in Section 3.1 and Section 3.2 following procedure is followed.

Starting from the specified initial conditions at time $\tau = 0$, we marched in time first and the velocity, temperature and concentration fields, and mean Nusselt and Sherwood numbers

were obtained at time $\tau + \Delta\tau$, for each X location, starting from the leading edge and marching downstream. At a particular location X and time τ, the iterative technique is applied as explained below.

a) The first iteration is started by guessing values for $U^{(o)}_{j+1,k,i+1}$, $V^{(o)}_{j+1,k,i+1}$, $\theta^{(o)}_{j+1,k,i+1}$ and $C^{(o)}_{j+1,k,i+1}$. These guesses are the values at the preceding step upstream (like $U_{j,k,i+1}$ etc.)

b) Taking $l = 0$ in equation (3.10), $U^{(1)}_{j+1,k,i+1}$ are calculated solving a set of (n) simultaneous equations. (solution procedure to be explained later.)

c) The continuity equation (3.9) again at $l = 0$ is solved for the transverse velocity component $V^{(1)}_{j+1,k+1,i+1}$, in the stepwise manner, working outward from the plate surface.

d) $\theta^{(1)}_{j+1,k,i+1}$ and $C^{(1)}_{j+1,k,i+1}$ are calculated at $l = 0$ from equations (3.11) and (3.12) respectively, solving sets of n simultaneous equations (solution procedure to be explained later.)

e) The entire procedure in steps b) to d) is repeated for $l = 1, 2, 3...$ and so on, until $U^{(l+1)}_{j+1,k,i+1}$ and $U^{(l)}_{j+1,k,i+1}$ agree to within a desired degree of accuracy ε, taken here as 10^{-3}. Similar degree of accuracy is set for $V_{j+1,k,i+1}$, $\theta_{j+1,k,i+1}$ and $C_{j+1,k,i+1}$.

It might be noted that this iterative procedure is a composite of Jacobi and Gauss-Siedel iterative techniques as extended to nonlinear equations.

Once iteration is complete at a particular location X and time τ, the instantaneous local Nusselt and Sherwood numbers are calculated solving equations (3.13a) and (3.13b) respectively.

Now another step ΔX downstream is taken and the process is repreated at the same τ. When the solution has been carried downstream as far as desired, the instantaneous mean Nusselt and Sherwood numbers are calculated solving the equations (3.16a) and (3.16b). Then another time step $\Delta\tau$ is taken, and again starting at the leading edge, the solution is marched spacewise downstream. The whole process is repeated as many times as necessary to determine the steady state solution such that $U_{j+1,k,i}$ and $U_{j+1,k,i+1}$ agree to within the desired degree of accuracy ε.

3.4 Solution Procedure :

At each iteration, the system of (n) simultaneous linear equations resulting from momentum equation (3.10) can be written in matrix form as

$$\begin{bmatrix} \beta_1^{(l)} & \Omega_1^{(l)} & & & & & \\ \alpha_2^{(l)} & \beta_2^{(l)} & \Omega_2^{(l)} & & & & \\ & \alpha_3^{(l)} & \beta_3^{(l)} & \Omega_3^{(l)} & & & \\ & & - & - & - & & \\ & & & - & - & - & \\ & & & & - & - & - \\ & & & & \alpha_{n-1}^{(l)} & \beta_{n-1}^{(l)} & \Omega_{n-1}^{(l)} \\ & & & & & \alpha_n^{(l)} & \beta_n^{(l)} \end{bmatrix} \begin{bmatrix} U_{j+1,1,i+1}^{(l+1)} \\ U_{j+1,2,i+1}^{(l+1)} \\ U_{j+1,3,i+1}^{(l+1)} \\ - \\ - \\ - \\ U_{j+1,n-1,i+1}^{(l+1)} \\ U_{j+1,n,i+1}^{(l+1)} \end{bmatrix} = \begin{bmatrix} \varphi_1^{(l)} \\ \varphi_2^{(l)} \\ \varphi_3^{(l)} \\ - \\ - \\ - \\ \varphi_{n-1}^{(l)} \\ (\varphi_n^{(l)} - \Omega_n^{(l)} U_\infty) \end{bmatrix} \qquad (3.17)$$

where

$$\alpha_k^{(l)} = -\frac{1}{(\Delta Y)^2} - \frac{V_{j+1,k,i+1}^{(l)}}{2(\Delta Y)}$$

$$\beta_k^{(l)} = \frac{1}{\Delta\tau} + \frac{2}{(\Delta Y)^2} + \frac{U_{j+1,k,i+1}^{(l)}}{\Delta X}$$

$$\Omega_k^{(l)} = -\frac{1}{(\Delta Y)^2} + \frac{V_{j+1,k,i+1}^{(l)}}{2(\Delta Y)}$$

$$\varphi_k^{(l)} = \frac{U_{j+1,k,i}}{\Delta\tau} + \frac{U_{j+1,k,i+1}^{(l)}\, U_{j,k,i+1}}{\Delta X} + \theta_{j+1,k,i+1}^{(l)}$$

$$+ N\, C_{j+1,k,i+1}^{(l)}.$$

Similarly the systems of (n) simultaneous linear equations for energy and species respectively resulting from equations (3.11) and (3.12) respectively are

$$\begin{bmatrix} \beta_1'^{(l)} & \Omega_1'^{(l)} & & & & & \\ \alpha_2'^{(l)} & \beta_2'^{(l)} & \Omega_2'^{(l)} & & & & \\ & \alpha_3'^{(l)} & \beta_3'^{(l)} & \Omega_3'^{(l)} & & & \\ & & - & - & - & & \\ & & & - & - & - & \\ & & & & - & - & - \\ & & & & \alpha_{n-1}'^{(l)} & \beta_{n-1}'^{(l)} & \Omega_{n-1}'^{(l)} \\ & & & & & \alpha_n'^{(l)} & \beta_n'^{(l)} \end{bmatrix} \times \begin{bmatrix} \theta_{j+1,1,i+1}^{(l+1)} \\ \theta_{j+1,2,i+1}^{(l+1)} \\ \theta_{j+1,3,i+1}^{(l+1)} \\ - \\ - \\ - \\ \theta_{j+1,n-1,i+1}^{(l+1)} \\ \theta_{j+1,n,i+1}^{(l+1)} \end{bmatrix} = \begin{bmatrix} (\varphi_1'^{(l)} - \alpha_1'^{(l)}) \\ \varphi_2'^{(l)} \\ \varphi_3'^{(l)} \\ - \\ - \\ - \\ \varphi_{n-1}'^{(l)} \\ \varphi_n'^{(l)} \end{bmatrix} \tag{3.18}$$

where

$$\alpha_k'^{(l)} = \frac{-1}{Pr(\Delta Y)^2} - \frac{V_{j+1,k,i+1}^{(l+1)}}{2(\Delta Y)}$$

$$\beta_k'^{(l)} = \frac{1}{\Delta\tau} + \frac{2}{Pr(\Delta Y)^2} + \frac{U_{j+1,k,i+1}^{(l+1)}}{\Delta X}$$

$$\Omega_k'^{(l)} = \frac{-1}{Pr(\Delta Y)^2} + \frac{V_{j+1,k,i+1}^{(l+1)}}{2(\Delta Y)}$$

$$\varphi_k'^{(l)} = \frac{\theta_{j+1,k,i}}{\Delta\tau} + \frac{U_{j+1,k,i+1}^{(l+1)}\,\theta_{j,k,i+1}}{\Delta X}$$

$$
\begin{bmatrix}
\beta_1''^{(l)} & \Omega_1''^{(l)} & & & & & \\
\alpha_2''^{(l)} & \beta_2''^{(l)} & \Omega_2''^{(l)} & & & & \\
 & \alpha_3''^{(l)} & \beta_3''^{(l)} & \Omega_3''^{(l)} & & & \\
 & & - & - & - & & \\
 & & & - & - & - & \\
 & & & & - & - & - \\
 & & & & \alpha_{n-1}''^{(l)} & \beta_{n-1}''^{(l)} & \Omega_{n-1}''^{(l)} \\
 & & & & & \alpha_n''^{(l)} & \beta_n''^{(l)}
\end{bmatrix}
\times
\begin{bmatrix}
C_{j+1,1,i+1}^{(l+1)} \\
C_{j+1,2,i+1}^{(l+1)} \\
C_{j+1,3,i+1}^{(l+1)} \\
- \\
- \\
- \\
C_{j+1,n-1,i+1}^{(l+1)} \\
C_{j+1,n,i+1}^{(l+1)}
\end{bmatrix}
=
\begin{bmatrix}
\varphi_1''^{(l)} - \alpha_1''^{(l)} \\
\varphi_2''^{(l)} \\
\varphi_3''^{(l)} \\
- \\
- \\
- \\
\varphi_{n-1}''^{(l)} \\
\varphi_n''^{(l)}
\end{bmatrix}
\tag{3.19}
$$

where

$$\alpha_k''^{(l)} = \frac{-1}{Sc(\Delta Y)^2} - \frac{V_{j+1,k,i+1}^{(l+1)}}{2(\Delta Y)}$$

$$\beta_k''^{(l)} = \frac{1}{\Delta\tau} + \frac{2}{Sc(\Delta Y)^2} + \frac{U_{j+1,k,i+1}^{(l+1)}}{\Delta X}$$

$$\Omega_k''^{(l)} = \frac{-1}{Sc(\Delta Y)^2} + \frac{V^{(l+1)}_{j+1,k,i+1}}{2(\Delta Y)}$$

$$\varphi_k''^{(l)} = \frac{C_{j+1,k,i}}{\Delta\tau} + \frac{U^{(l+1)}_{j+1,k,i+1}\; C_{j,k,i+1}}{\Delta X} .$$

Solution of system (3.17) to (3.19) by the inversion method is highly expensive even on the present-day computers since the number of steps required is of order $(n)^3$ and the storage requirement is of order $(n)^2$. As the matrix of coefficients in all the systems (3.17) to (3.19) is tridiagonal a subroutine TRIDIA [16] is used which works efficiently.

3.5. Convergence and Relaxation :

In some cases it is desirable to either overrelax or underrelax the iterative procedure in order to speed up or slow down the changes from iteration to iteration, in the values of dependent variables. Under relaxation is very useful for nonlinear problems. It is used to avoid divergence in the iterative solution of strongly nonlinear problems.

In the present problem after preliminary calculations it was observed that the iterative procedure does not converge unless some underrelaxation is employed. The following relaxation procedure was used.

In the direct iterative procedure the quantities $U^{(l)}_{j+1,k,i+1}$, $\theta^{(l)}_{j+1,k,i+1}$ and $C^{(l)}_{j+1,k,i+1}$ appearing in the momentum, energy and chemical species equations respectively take on the values of $U^{(l+1)}_{j+1,k,i+1}$, $\theta^{(l+1)}_{j+1,k,i+1}$, $C^{(l+1)}_{j+1,k,i+1}$ respectively after each iteration. In the relaxation procedure

employed the values of U's, θ's and C's are modified as

$$
\begin{aligned}
U^{(l)}_{j+1,k,i+1} &\longleftarrow U^{(l)}_{j+1,k,i+1} + \lambda_U\left(U^{(l+1)}_{j+1,k,i+1} - U^{(l)}_{j+1,k,i+1}\right)\\
\theta^{(l)}_{j+1,k,i+1} &\longleftarrow \theta^{(l)}_{j+1,k,i+1} + \lambda_\theta\left(\theta^{(l+1)}_{j+1,k,i+1} - \theta^{(l)}_{j+1,k,i+1}\right) \qquad (3.20)\\
C^{(l)}_{j+1,k,i+1} &\longleftarrow C^{(l)}_{j+1,k,i+1} + \lambda_C\left(C^{(l+1)}_{j+1,k,i+1} - C^{(l)}_{j+1,k,i+1}\right)
\end{aligned}
$$

where λ_U , λ_θ and λ_C are the underrelaxation factors. The factors found useful in the preliminary calculations were $\lambda_U = \lambda_\theta = \lambda_C = 0.3$. However as we go downstream they were increased to 1.0. They depend also on the value of U_∞. Eventhough optimum values were not used effort was made to choose values by trial and error such that computational time was minimized.

3.6. Selection of Step Size :

No quantitative statements can be made about the choice of step sizes. However smaller step sizes are preferred in regions of more rapidly changing velocity, temperature and concentration profiles, in order to reduce the error. Thus a fine mesh size is required close to the plate surface and close to the leading edge. Effects of singularity at the leading edge in the boundary layer problem can be confined to a small region close to the leading edge by selecting small mesh size ΔX until the profiles smooth out somewhat.

Since implicit formulation is used, there is no restriction on the selection of step sizes $\Delta\tau$ and ΔX, from the point of view of stability. However, since at small time heat is transferred by conduction only and mass is transferred by diffusion only, and Nu_m and Sh_m are inversely proportional to $\sqrt{\tau}$ in that region, small time-step sizes are used initially and then time-step size is increased to reduce the computational time. Following time-step sizes were used.

<u>for $U_\infty = 0.0$ and 1.0</u>

$\Delta\tau = 0.05 \qquad (0 \leq \tau \leq 0.4)$

$\Delta\tau = 0.2 \qquad (0.4 \leq \tau \leq \text{steady state } \tau)$

and

<u>for $U_\infty = 10.0$</u>

$\Delta\tau = 0.05 \qquad (0 \leq \tau \leq 0.2)$

$\Delta\tau = 0.2 \qquad (0.2 \leq \tau \leq \text{steady state } \tau)$

In order to reduce the number of equations to be solved, which can effect a considerable saving in computer time, and to keep round-off error accumulated in solving large number of simultaneous equations to a minimum, it is necessary to use variable mesh size. Fine mesh size in regions of rapidly varying velocities and relatively coarse mesh size in regions of slowly varying velocities is called for. Variable mesh sizes employed in Y-direction were as follows :

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<u>for $U_\infty = 0.0$ and 1.0</u>

$$\Delta\tau = 0.05 \qquad (0 \leq \tau \leq 0.4)$$

$$\Delta\tau = 0.2 \qquad (0.4 \leq \tau \leq \text{steady state } \tau)$$

and

<u>for $U_\infty = 10.0$</u>

$$\Delta\tau = 0.05 \qquad (0 \leq \tau \leq 0.2)$$

$$\Delta\tau = 0.2 \qquad (0.2 \leq \tau \leq \text{steady state } \tau)$$

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<u>for $U_\infty = 0.0$</u>

$$\Delta Y = 0.05 \quad (0 \leq Y \leq 0.5)$$

$$\Delta Y = 0.15 \quad (0.5 \leq Y \leq 2.0)$$

$$\Delta Y = 0.30 \quad (2.0 \leq Y \leq 8.0)$$

$$\Delta Y = 0.50 \quad (8.0 \leq Y \leq 17.0)$$

<u>for $U_\infty = 1.0$</u>

$$\Delta Y = 0.03 \quad (0 \leq Y \leq .30$$

$$\Delta Y = 0.10 \quad (.30 \leq Y \leq 2.0)$$

$$\Delta Y = 0.25 \quad (2.0 \leq Y \leq 4.50)$$

$$\Delta Y = 0.50 \quad (4.50 \leq Y \leq 11.50)$$

<u>for $U_\infty = 10.0$</u>

$$\Delta Y = 0.01 \quad (0 \leq Y \leq 0.1)$$

$$\Delta Y = 0.04 \quad (0.1 \leq Y \leq 0.5)$$

$$\Delta Y = 0.10 \quad (0.5 \leq Y \leq 1.5)$$

$$\Delta Y = 0.50 \quad (1.5 \leq Y \leq 6.0)$$

The variable mesh technique provides no difficulty for forward or backward first order differences of error o(h), where h is the mesh size. However, when central differences, either first or second order, are used as in the transverse direction; difficulties arise at the point of mesh size change. To alleviate such a difficulty, consider the mesh size change

from a smaller step size $\Delta Y1$ to a larger step size $\Delta Y2$ at the Y-location $k = p$ in fig. 3.1. Application of the central difference form for the first or second derivative at $k = p$ requires the value at ficticious $k = q$. This value is found by passing a parabola through the values at $k = (p-1), p$ and $(p+1)$ to yield

$$Q_{j+1,q} = \frac{\Phi_R - 1}{\Phi_R + 1} Q_{j+1,p-1} + 2(1-\Phi_R) Q_{j+1,p} + 2 \frac{\Phi_R^2}{1+\Phi_R} Q_{j+1,p+1} \tag{3.21}$$

where Q is the dependent variable at some instant τ. The derivatives at $k = p$ are then approximated as

$$\left.\frac{\partial Q}{\partial Y}\right|_{k=p} = \frac{Q_{j+1,q} - Q_{j+1,p-1}}{2(\Delta Y1)} \tag{3.22a}$$

$$\left.\frac{\partial^2 Q}{\partial Y^2}\right|_{k=p} = \frac{Q_{j+1,q} - 2\, Q_{j+1,p} + Q_{j+1,p-1}}{(\Delta Y1)^2} \tag{3.22b}$$

where $Q_{j+1,q}$ is given by equation (3.21) and Φ_R is the ratio of small step size to large step size.

At the points of mesh size change equations (3.22a) and (3.22b) are used in the momentum, energy, and species equations and at these points modified equations are rewritten, as they involve central differences in transverse direction.

The difference representation for the continuity equation involves only first order forward differences in the transverse direction. Hence no such modification in equation (3.9) is required. It is ensured that the proper mesh size is used in the appropriate region.

Modified equations at points of mesh size change k = p are

Momentum Equation :

$$\frac{U^{(l+1)}_{j+1,p,i+1} - U_{j+1,p,i}}{\Delta\tau} + U^{(l)}_{j+1,p,i+1}\left(\frac{U^{(l+1)}_{j+1,p,i+1} - U_{j,p,i+1}}{\Delta X}\right)$$

$$+ V^{(l)}_{j+1,p,i+1}\left|\frac{\frac{\Phi_R-1}{\Phi_R+1} U^{(l+1)}_{j+1,p-1,i+1} + 2(1-\Phi_R) U^{(l+1)}_{j+1,p,i+1} + 2\frac{\Phi_R^2}{1+\Phi_R} U^{(l+1)}_{j+1,p+1,i+1} - U^{(l+1)}_{j+1,p-1,i+1}}{2(\Delta Y)}\right| \tag{3.23}$$

$$= \frac{\left[\frac{\Phi_R-1}{\Phi_R+1} U^{(l+1)}_{j+1,p-1,i+1} + 2(1-\Phi_R) U^{(l+1)}_{j+1,p,i+1} + 2\frac{\Phi_R^2}{1+\Phi_R} U^{(l+1)}_{j+1,p+1,i+1} - 2U^{(l+1)}_{j+1,p,i+1} + U^{(l+1)}_{j+1,p-1,i+1}\right]}{(\Delta Y)^2}$$

$$+ \theta^{(l)}_{j+1,p,i+1} + N\,C^{(l)}_{j+1,p,i+1}$$

which is written in the simplified form as

$$\left[\frac{\left(\frac{\Phi_R-1}{\Phi_R+1} - 1\right) V^{(l)}_{j+1,p,i+1}}{2(\Delta Y)} - \frac{\left(1 + \left(\frac{\Phi_R-1}{\Phi_R+1}\right)\right)}{(\Delta Y)^2}\right] U^{(l+1)}_{j+1,p-1,i+1}$$

$$+\left[\frac{1}{\Delta\tau} + \frac{U^{(l)}_{j+1,p,i+1}}{\Delta X} + \frac{2(1-\Phi_R)\, V^{(l)}_{j+1,p,i+1}}{2(\Delta Y)} + \frac{2\Phi_R}{(\Delta Y)^2}\right] U^{(l+1)}_{j+1,p,i+1}$$

$$+\left[\left(\frac{2\Phi_R^2}{1+\Phi_R}\right)\left(\frac{-1}{(\Delta Y)^2} + \frac{V^{(l)}_{j+1,p,i+1}}{2(\Delta Y)}\right)\right] U^{(l+1)}_{j+1,p+1,i+1}$$

$$= \frac{U_{j+1,p,i}}{\Delta\tau} + \frac{U^{(l)}_{j+1,p,i+1} U_{j,p,i+1}}{\Delta X} + \theta^{(l)}_{j+1,p,i+1} + NC^{(l)}_{j+1,p,i+1} \tag{3.24}$$

Similarly energy and concentration equations are written as

$$\left[\frac{-(1+(\frac{\Phi_R-1}{\Phi_R+1}))}{Pr\,(\Delta Y)^2} - \frac{(1-(\frac{\Phi_R-1}{\Phi_R+1}))\,V^{(l+1)}_{j+1,p,i+1}}{2(\Delta Y)}\right] \theta^{(l+1)}_{j+1,p-1,i+1}$$

$$+ \left[\frac{1}{\Delta\tau} + \frac{2\Phi_R}{Pr(\Delta Y)^2} + \frac{2(1-\Phi_R)\,V^{(l+1)}_{j+1,p,i+1}}{2(\Delta Y)} + \frac{U^{(l+1)}_{j+1,p,i+1}}{\Delta X}\right] \theta^{(l+1)}_{j+1,p,i+1}$$

$$+ \left[\frac{2\Phi_R^2}{(1-\Phi_R)} \left(\frac{-1}{(\Delta Y)^2} + \frac{V^{(l+1)}_{j+1,p,i+1}}{2(\Delta Y)}\right)\right] \theta^{(l+1)}_{j+1,p+1,i+1}$$

$$= \frac{\theta_{j+1,p,i}}{\Delta\tau} + \frac{U^{(l+1)}_{j+1,p,i+1}\,\theta_{j,p,i+1}}{\Delta X} \tag{3.25}$$

$$\left[\frac{-(1+(\frac{\Phi_R-1}{\Phi_R+1}))}{Sc(\Delta Y)^2} - \frac{(1-(\frac{\Phi_R-1}{\Phi_R+1}))\,V^{(l+1)}_{j+1,p,i+1}}{2(\Delta Y)}\right] C^{(l+1)}_{j+1,p-1,i+1}$$

$$+ \left[\frac{1}{\Delta\tau} + \frac{2\Phi_R}{Sc(\Delta Y)^2} + \frac{2(1-\Phi_R)\,V^{(l+1)}_{j+1,p,i+1}}{2(\Delta Y)} + \frac{U^{(l+1)}_{j+1,p,i+1}}{\Delta X}\right] C^{(l+1)}_{j+1,p,i+1}$$

$$+ \left[\frac{2\Phi_R^2}{1+\Phi_R} \left(\frac{-1}{(\Delta Y)^2} + \frac{V^{(l+1)}_{j+1,p,i+1}}{2(\Delta Y)}\right)\right] C^{(l+1)}_{j+1,p+1,i+1}$$

$$= \frac{C_{j+1,p,i}}{\Delta\tau} + \frac{U^{(l+1)}_{j+1,p,i+1}\,C_{j,p,i+1}}{\Delta X}\,, \tag{3.26}$$

A constant mesh size of $\Delta X = 0.02$ was used with 50 steps in X-direction, till the upper edge of the plate ($X = 1.0$) was reached. The computational procedure took slightly over 3 minutes of CPU time for $U_\infty = 0.0$, $Sc = 0.2$ and $N = 2.0$ which reduced further with $Sc = 2.0$ and $N = 0.0$. For $U_\infty = 1.0$ and 10.0 CPU time required was around 1 minute. This time could have been shortened further if a variable mesh sizes were used in the X-direction, but in the interest of simplicity in computer programming constant mesh size was used in the X-direction.

Chapter 4

RESULTS AND DISCUSSION

4.1 Limiting Checks :

In order to assess the accuracy of the numerical procedure, several cases were solved for pure natural convection ($U_\infty = 0.0$) and resulting results were compared with those of Callahan and Marner [13]. Excellent agreement was obtained for steady state velocity, temperature and concentration profiles at $X = 1.0$ for $Pr = 1.0$, $Sc = 0.7$, 7.0 and $N = 0.0$, 1.0, 2.0. Transient Nusselt and Sherwood numbers for free convection ($U_\infty = 0.0$) were also compared with the results of Callahan and Marner [13] for $N = 0,0$, 2.0 and $Pr = 1.0$ and for various values of Sc. Excellent agreement was found over the entire time interval. Based on these comparisons, it is felt that the present numerical procedure can predict both transient and steady-state results quite accurately. Below we present results for $Pr = 0.7$, $Sc = 0.2$ and 2.0, $N = 0.0$ and 2.0, and $U_\infty = 0,1$ and 10. The Pr and Sc values are representative of a large number of gases.

4.2 Velocity, Temperature and Concentration Profiles :

Fig. 4.1 shows the typical development of transient dimensionless X-component of velocity U at $Pr = 0.7$, $Sc = 0.2$ and $N = 2.0$ covering conditions ranging from almost pure forced convection ($U_\infty = 10.0$), combined flow with almost equally strong

natural and forced convection contributions ($U_\infty = 1.0$), and pure natural convection ($U_\infty = 0.0$). The profiles presented are those at the upper edge of the plate i.e. at $X = 1.0$. Numerical values are listed in tables 4.4 to 4.6. For free convection case i.e. at $U_\infty = 0.0$, it is observed that the velocity increases continuously with time until at $\tau \simeq 2.40$ it reaches a maximum value and then it decreases slightly to the steady-state value at $\tau \simeq 2.80$ (with degree of accuracy of $\varepsilon = 10^{-3}$). The difference between the temporal maximum in the velocity profile and the steady-state value, however, is quite small and is imperceptible if shown on the figure. That is why it is not shown.

The phenomenon of temporal maximum in the velocity profile is somewhat surprising. It has been observed and discussed by several investigators for the problem of transient free convection on a vertical plate in the absence of mass transfer. Siegel [18] based on an approximate integral analysis, was apparently the first to predict such a behaviour. Later analysis by Gebhart [19], Hellums and Churchill [12] and Kleppe and Marner [20] all confirmed the findings of Siegel. Callahan and Marner [13] predicted such a phenomenon for the more complex problem involving simultaneous effect of heat and mass transfer in which N and Sc in addition to Pr, are the controlling parameters. The maximum velocity apparently occurs when the buoyancy forces in the fluid are largest, and it is clear that both the magnitude of the maximum velocity and the time at which it occurs are functions of these three parameters. However

the phenomenon of temporal maximum is not observed for combined free and forced convection ($U_\infty = 1.0$) and almost pure forced convection ($U_\infty = 10.0$). In these cases forced convection dominates the natural convection.

The time required to reach steady state velocity decreases as U_∞ is increased. This is due to gradual masking of the natural convection by the forced convection. Figs. 4.1 and 4.2 show that for the same value of N, Pr and Sc the velocity boundary thickness decreases with increasing U_∞. This is also expected and is in line with the general behaviour of laminar boundary layers. Numerical values are listed in tables 4.1 to 4.6.

The effect of parameter N on the steady state velocity profile, again at $X = 1.0$, is shown in figs. 4.3 and 4.4 for $Pr = 0.7$ and $Sc = 0.2$ and 2.0 respectively for values of $U_\infty = 0.0$, 1.0 and 10.0. Clearly, the contribution of mass diffusion to the buoyancy force increases the maximum velocity significantly for low values of U_∞, both 0.0 and 1.0, though the increase is comparatively less for $U_\infty = 1.0$, for values of $U_\infty = 10.0$ and more (forced flow regime) this increase is almost insignificant.

A comparison of figs. 4.3 and 4.4 shows that the effect of the contribution of mass diffusion to the buoyancy force decreases for higher Schmidt numbers. This may be attributed to the fact that the rate of mass transfer in the fluid, which

in turn influences the buoyancy force, decreases as the Schmidt number increases. In the forced convection regime, however, increase in Schmidt number does not affect the maximum velocity as expected. Numerical values for the same are listed in tables 4.1 to 4.12.

Figs. 4.5 and 4.6 depict the development of transient dimensionless temperature and concentration profiles at $X = 1.0$ for $Pr = 0.7$, $Sc = 0.2$ and 2.0 respectively and $N = 2.0$ for pure natural convection case i.e. $U_\infty = 0.0$. Numerical values of temperature and concentration distributions are listed in tables 4.4 and 4.10. The temperature and concentration distributions in fig. 4.5, much like the velocity profile in fig. 4.1, increase to a maximum value at $\tau \simeq 1.20$ and then decrease slightly to the steady state value at $\tau \simeq 2.80$. Similarly in fig. 4.6 for $Sc = 2.0$ the temperature and concentration distributions increase to a maximum value at $\tau \simeq 1.6$ and then decrease slightly to the steady state value at $\tau \simeq 2.80$. Though the maximum temperature and concentration profiles for $Sc = 0.2$ occur sooner than that for $Sc = 2.0$, the time at which the respective temperature and concentration profiles reach their steady state value is the same.* Experimental data obtained by Goldstein R.J. and Eckert [21] and Klei for a vertical flat plate subjected to a step change in heat flux also verified this interesting overshoot phenomenon. As in the case of velocity profile the parameters Sc, N and Pr influence the

* This does not imply that their transient behaviour is identical.

extent of overshoot in the temperature and concentration profiles and the instant at which this maximum occurs. However this overshoot phenomenon is not observed when forced convection in addition to natural convection comes into play.

A comparison of the transient concentration profiles in figs. 4.5 and 4.6 shows that the concentration boundary layer for pure natural convection ($U_\infty = 0.0$) is considerably thinner for Sc = 2.0 than that for Sc = 0.2. But for $U_\infty = 10.0$ i.e. when forced convection is dominant, this difference in the concentration boundary layer thickness at Sc = 0.2 and 2.0 is much less as seen from fig. 4.7 and table 4.6. Similar conclusions can be drawn for N = 0.0 by comparing figs. 4.9 and 4.10.

Figs. 4.8 and 4.9 show the steady state temperature and concentration profiles at X = 1.0 for Pr = 0.7, Sc = 2.0 and N = 2.0 and 0.0 respectively for all three values of U_∞. From figs. 4.5, 4.6, 4.8 and 4.9 comparing transient and steady state temperature and concentration profiles for various values of N, Sc and U_∞, it is observed that the concentration boundary layer is thicker than thermal boundary layer for Sc = 0.2 while opposite is true for Sc = 2.0*. This difference in thickness decreases as U_∞ increases. Fig. 4.10 shows the steady state concentration profiles at X = 1.0, Pr = 0.7 and Sc = 0.2 with N and U_∞ as parameters. Note that the effect of N on the concentration profile decreases as U_∞ is increased. At $U_\infty = 10.0$,

* The correct parameter is the Lewis number, Le = Sc/Pr, but Pr is held constant here.

both values of N yield almost the same concentration profile. This is expected since buoyancy forces are insignificant at such a high forced flow condition. Numerical values are listed in tables4.1 to 4.12.

4.3. Nusselt and Sherwood Numbers :

Transient mean Nusselt and Sherwood numbers are shown in figs. 4.11, 4.12 and 4.13 for Pr = 0.7, Sc = 0.2 and 2.0, N = 0.0 and 2.0, and U_∞ = 0.0, 1.0 and 10.0. Numerical values are listed in tables 4.13 and 4.14. Initially for any set of parameters values of mean Nusselt number and Sherwood number are high but they drop drastically with time and approach steady state values. During this initial transient U_∞ has negligible influence on both Nu_m and Sh_m due to the fact that heat is transferred by conduction only and mass is transferred by diffusion only during this regime. As the buoyancy forces due to mass transfer and thermal convection increase, the velocity increases sufficiently for N and U_∞ to influence the solution. Because of the overshoot phenomenon observed in the temperature and concentration profiles (i.e. these profiles reaching a maximum before steady-state conditions are reached), a transient minimum is observed in both the Nusselt and Sherwood numbers,for pure natural convection. The difference between the temporal minimum and the steady state value, however, is quite small, and in most cases is nearly imperceptible on the figures. (See the inset in fig. 4.12). However for U_∞ = 1.0 and

10.0 where forced convection is predominant such a temporal minimum is not observed since the overshoot phenomenon is not present.

Both the mean Nusselt and the mean Sherwood numbers show a slight dependence on Sc and U_∞ as far as the time required to reach steady state conditions is concerned, i.e., as Sc increases and as U_∞ decreases the time decreases. However, the time required to reach steady-state conditions is virtually insensitive to the parameter N.

As the Schmidt number increases from 0.2 to 2.0 with Prandtl no. fixed at 0.7 and N fixed at 2.0, Sherwood number increases substantially but the Nusselt number is only moderately affected.

From figs. 4.14 and 4.15 it is noted that an increase in N results in higher Nusselt and Sherwood numbers. That is, an increase in the buoyancy force due to mass transfer results in an increase in the rates of heat and mass transfer. The effect of parameter N on the Nusselt number becomes less pronounced with increasing Schmidt number but its effect on Sherwood number is opposite and to a small extent. This behaviour can be explained as follows. It was observed that an increase in Schmidt number decreases the concentration boundary layer thickness. However, the thermal boundary layer thickness is relatively less sensitive to an increase in the Schmidt number for fixed values of Pr and U_∞ eventhough the conservation equations are coupled. Thus with

an increase in Schmidt number the concentration boundary layer thickness becomes thinner than the thermal boundary layer thickness. Hence the influence of parameter N on Nu_m reduces with increasing, Schmidt number, as the buoyancy effect due to mass transfer are diminished in the thermal boundary layer. On the contrary thermal buoyancy effects become less important as compared to buoyancy effects due to mass transfer in the thinner concentration boundary layer. Hence the influence of N on Sh_m increases with increased Schmidt number.

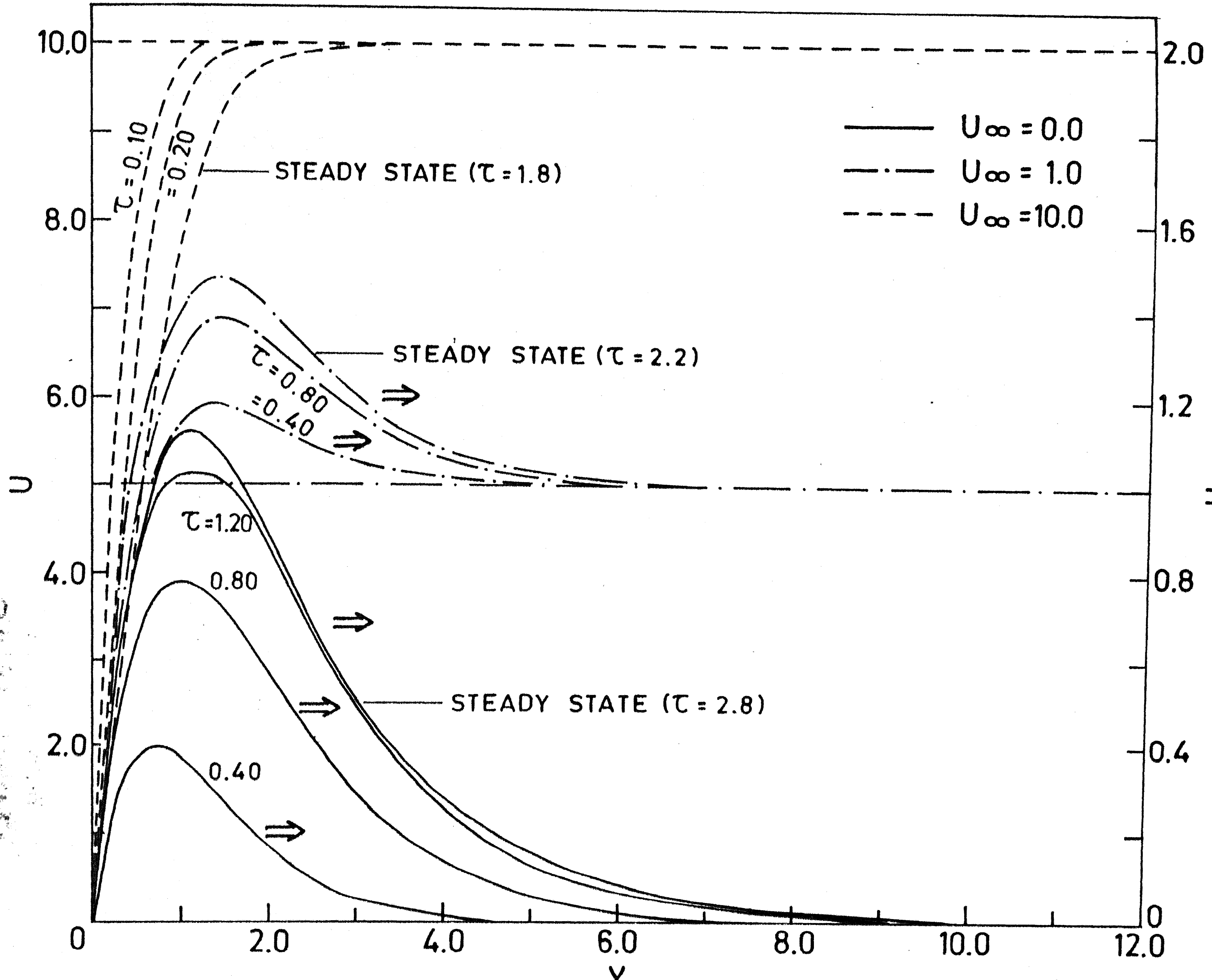

Fig. 4.1 Transient velocity profiles at X=1.0 for Pr=0.7, Sc=0.2, N=2.0

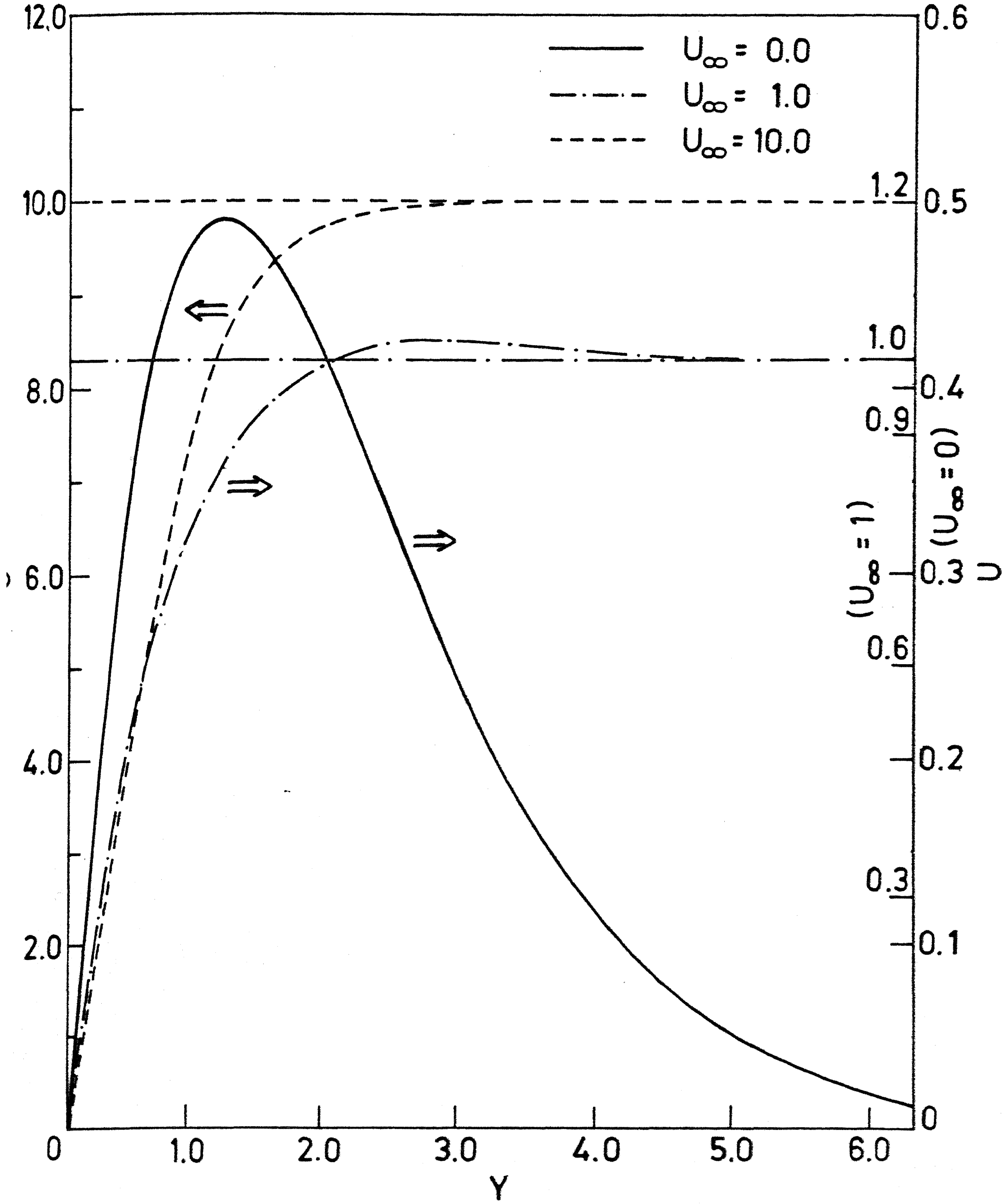

Fig. 4.2 Steady state velocity profiles at X = 1.0 for Pr = 0.7, Sc = 0.2, N = 0.0

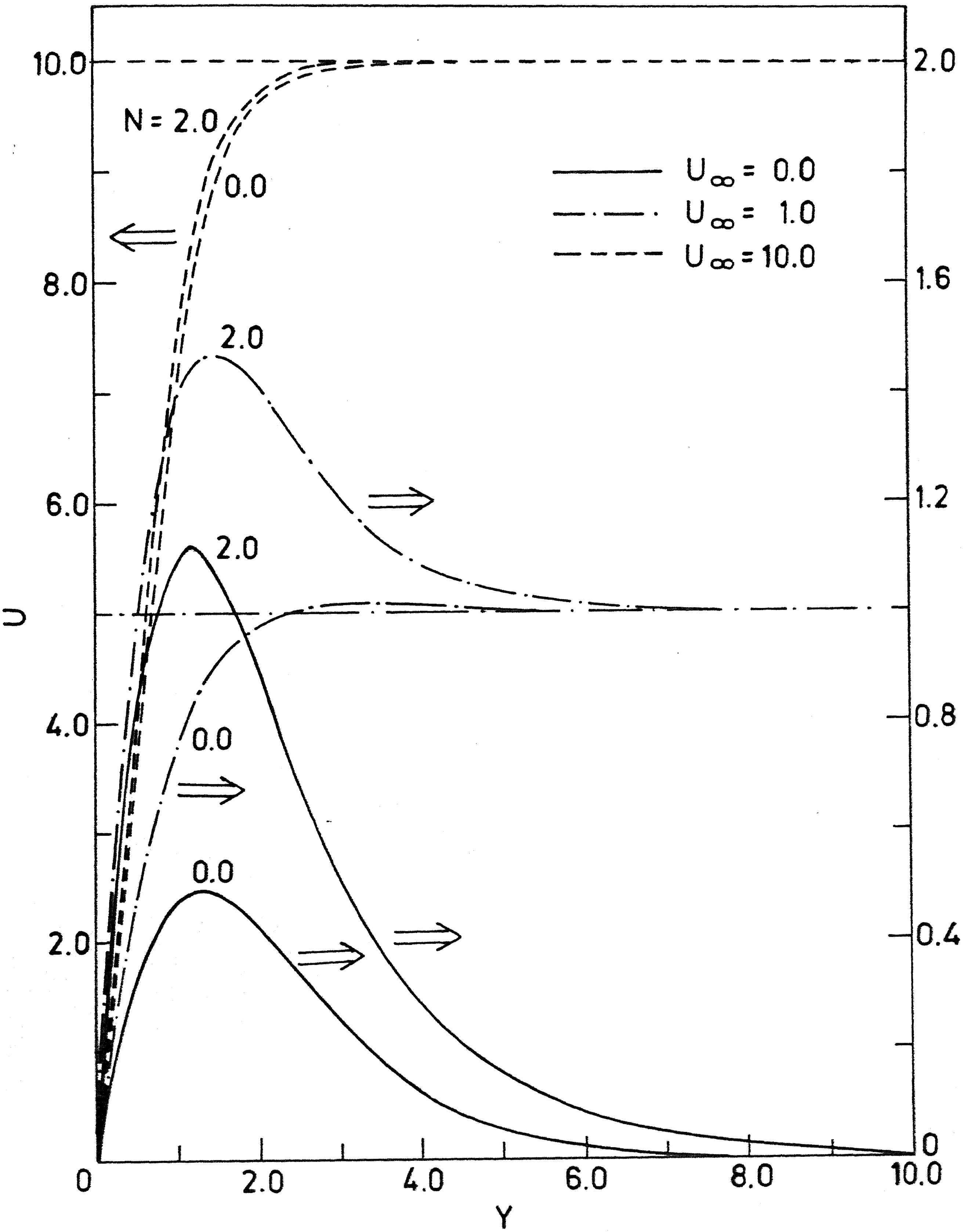

Fig. 4.3 Steady state velocity profiles at X = 1.0 as a function of N for Pr = 0.7, Sc = 0.2

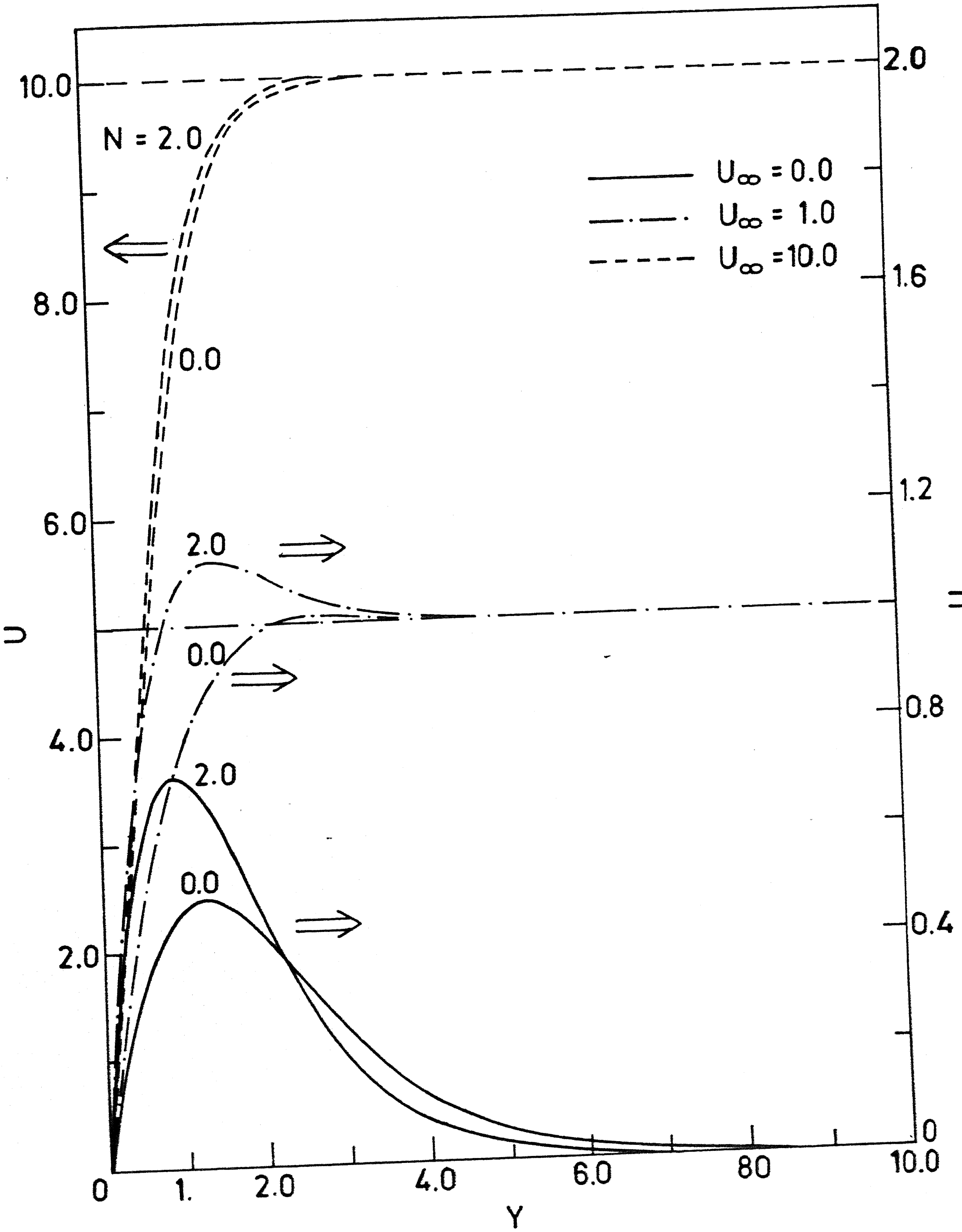

Fig. 4.4 Steady state velocity profiles at X = 1.0 as a function of N for Pr = 0.7, Sc = 2.0

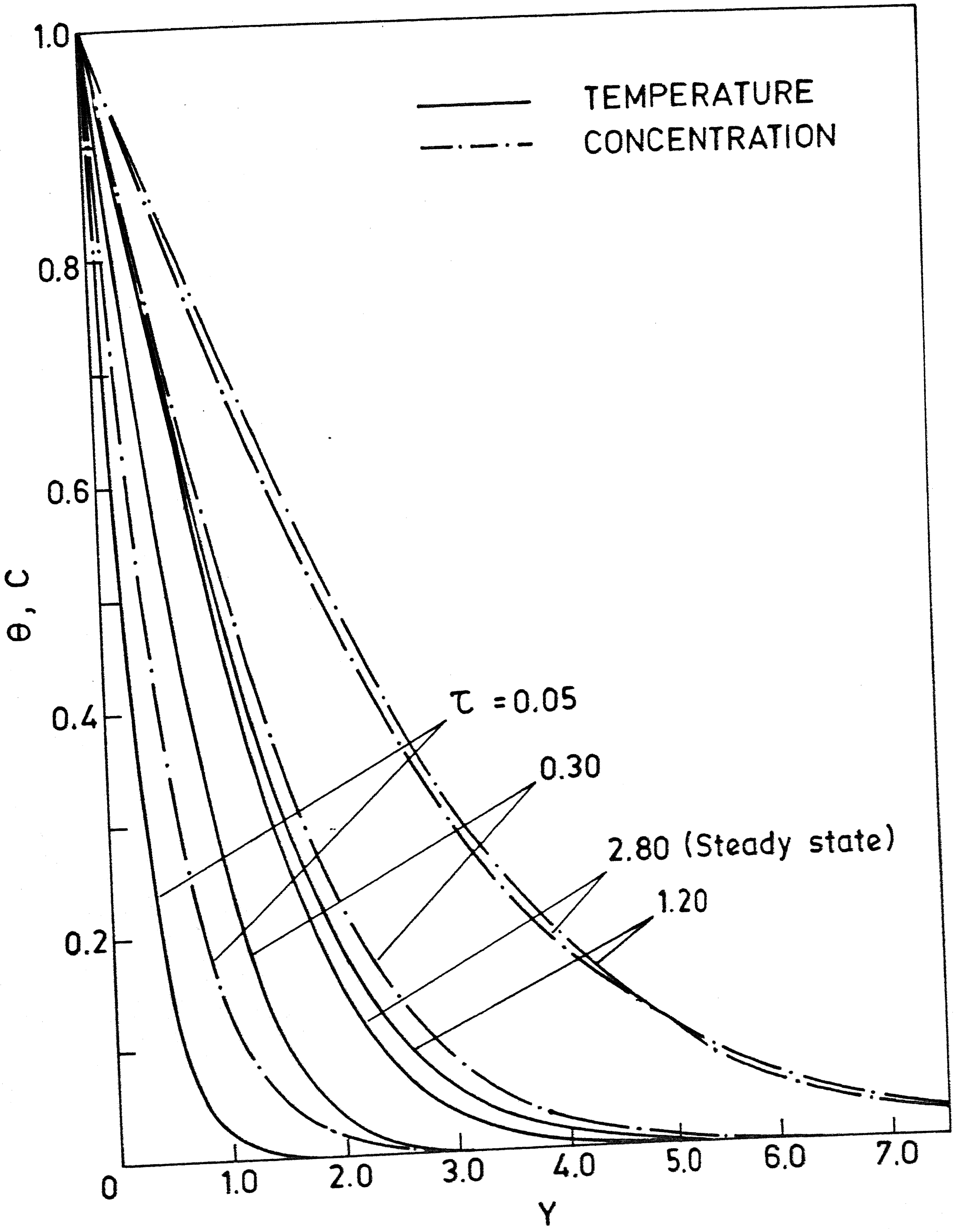

Fig. 4.5 Transient temperature and concentration profiles at X = 1.0 for Pr = 0.7, Sc = 0.2, N = 2.0 and $U_{\infty} = 0.0$

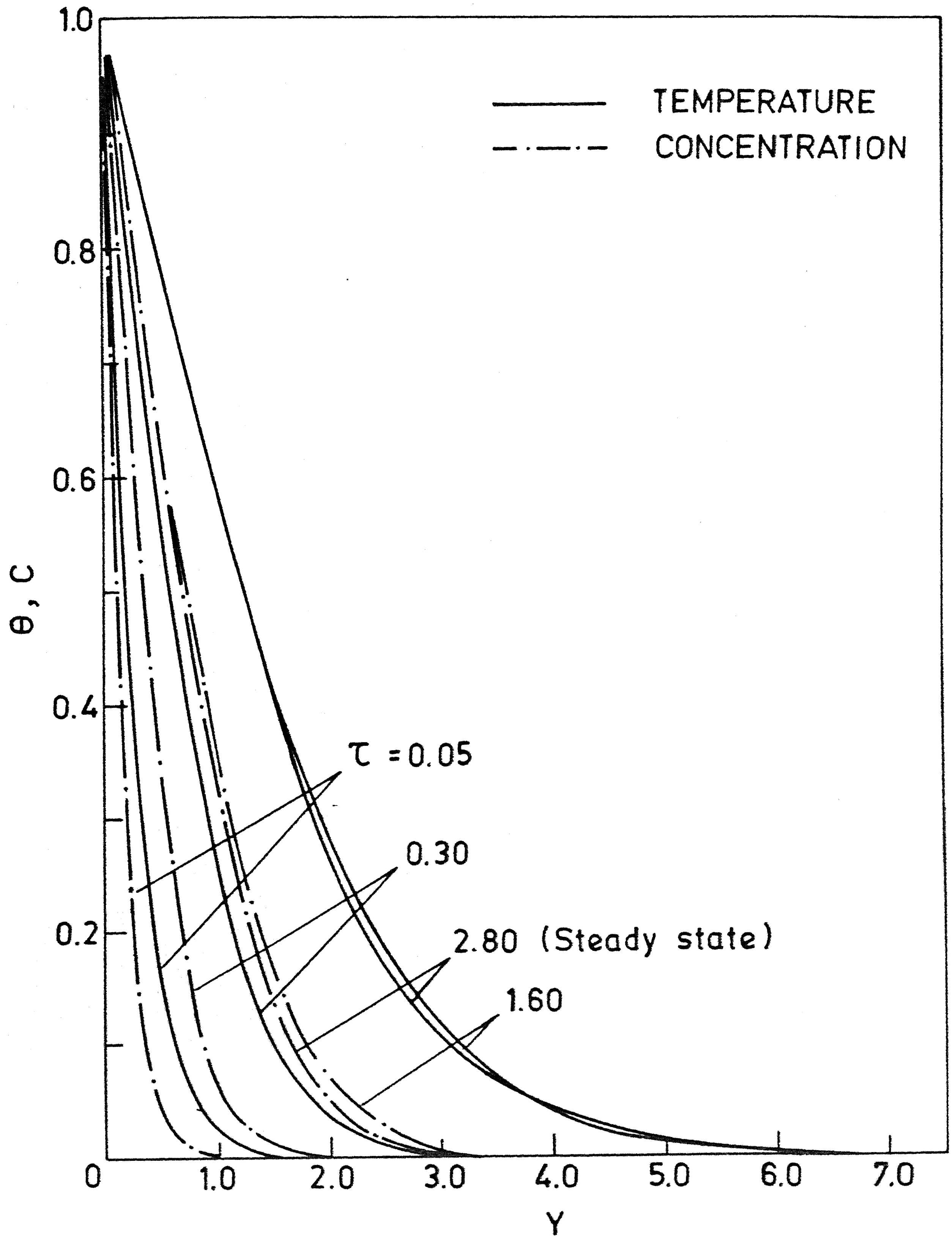

Fig. 4.6 Transient temperature and concentration profiles at $X = 1.0$ for $Pr = 0.7$, $Sc = 2.0$, $N = 2.0$ and $U_{\infty} = 0.0$

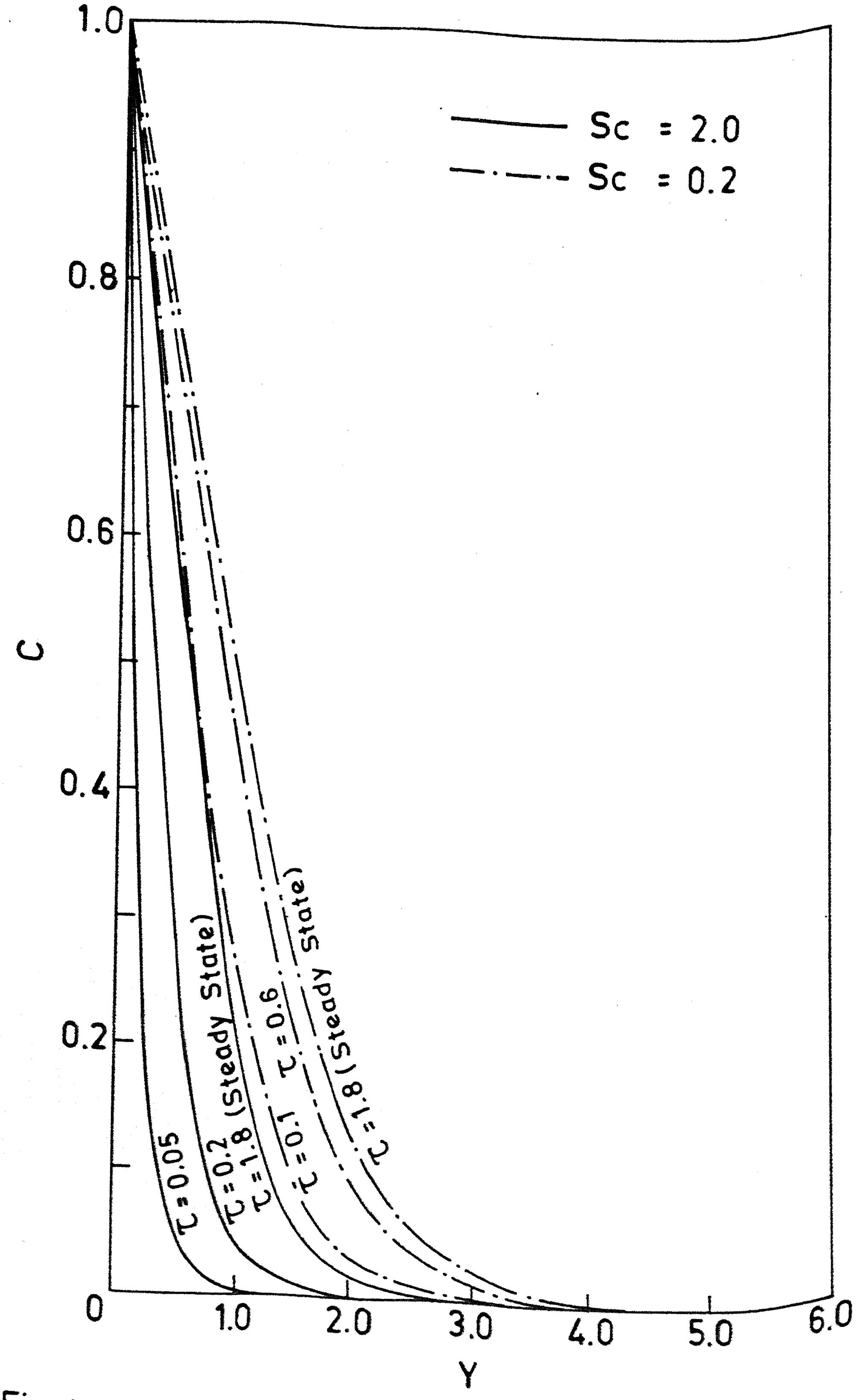

Fig. 4.7 Transient concentration profiles at $X = 1.0$ for $Pr = 0.7$, $N = 2.0$, $Sc = 0.2$ and 2.0 and $U_\infty = 10.0$

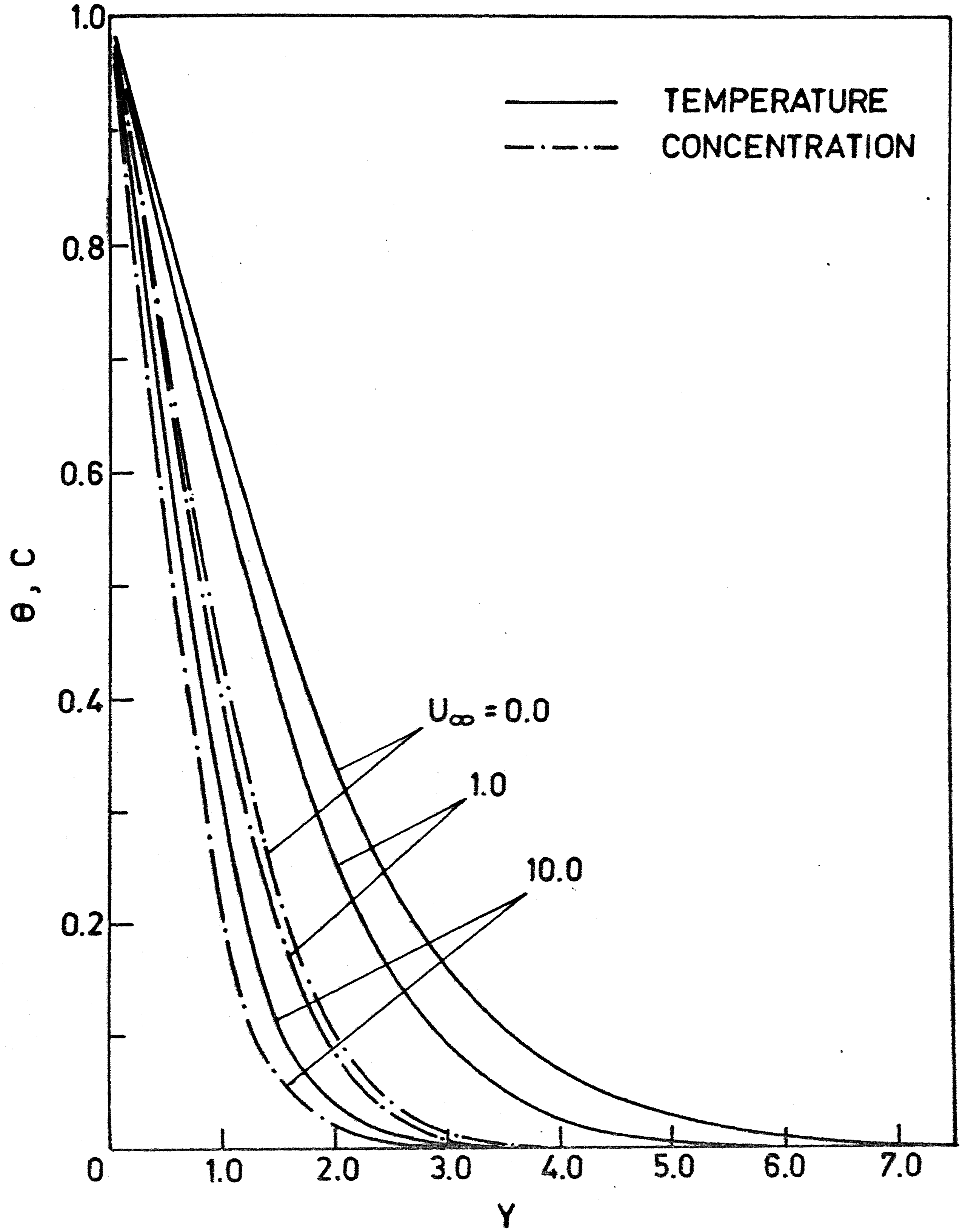

Fig. 4.9 Steady state temperature and concentration profiles at $X = 1.0$ for $Pr = 0.7$, $Sc = 2.0$, $N = 0.0$

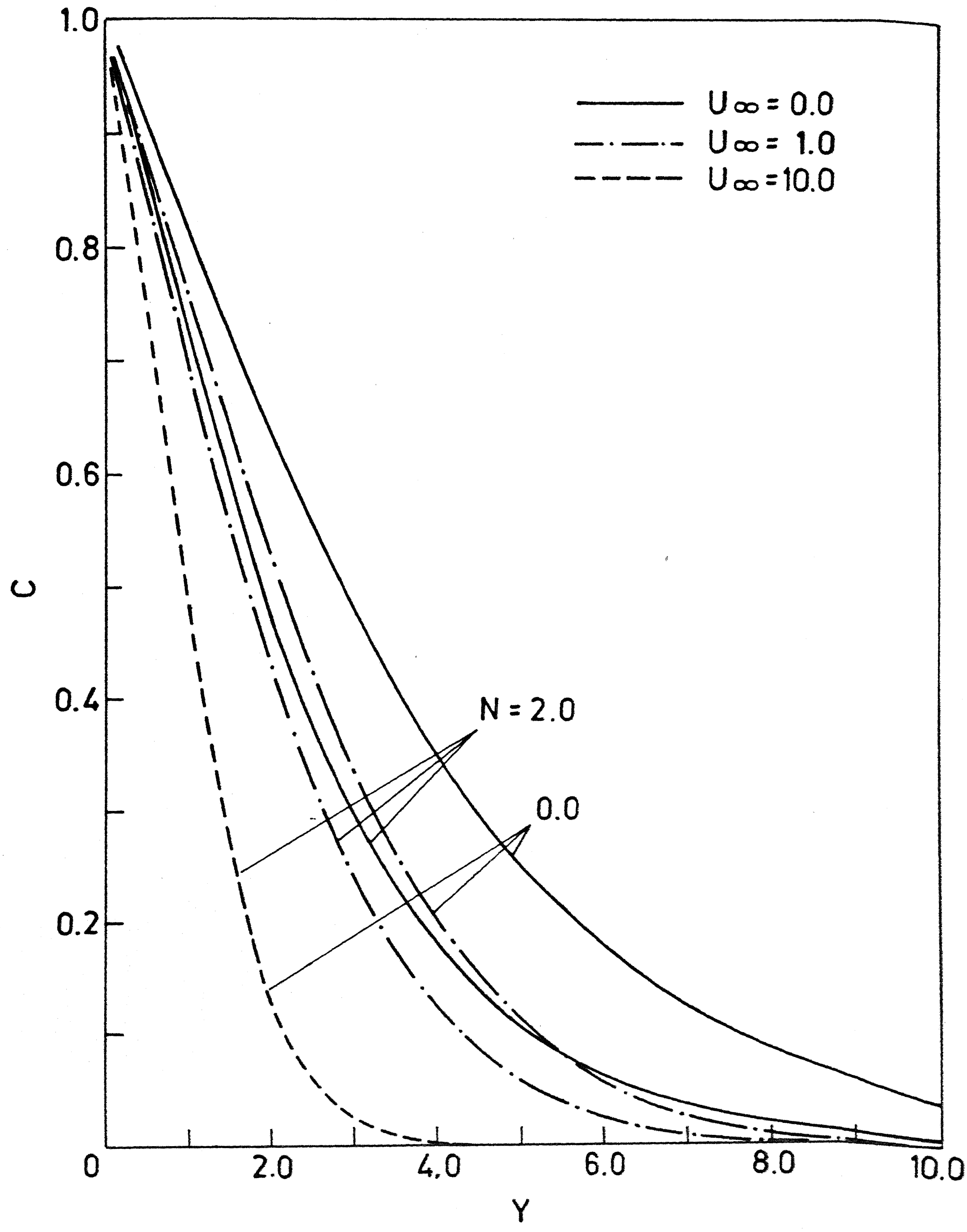

Fig. 4.10 Steady state concentration profiles at $X = 1.0$ as a function of N for $Pr = 0.7$, $Sc = 0.2$

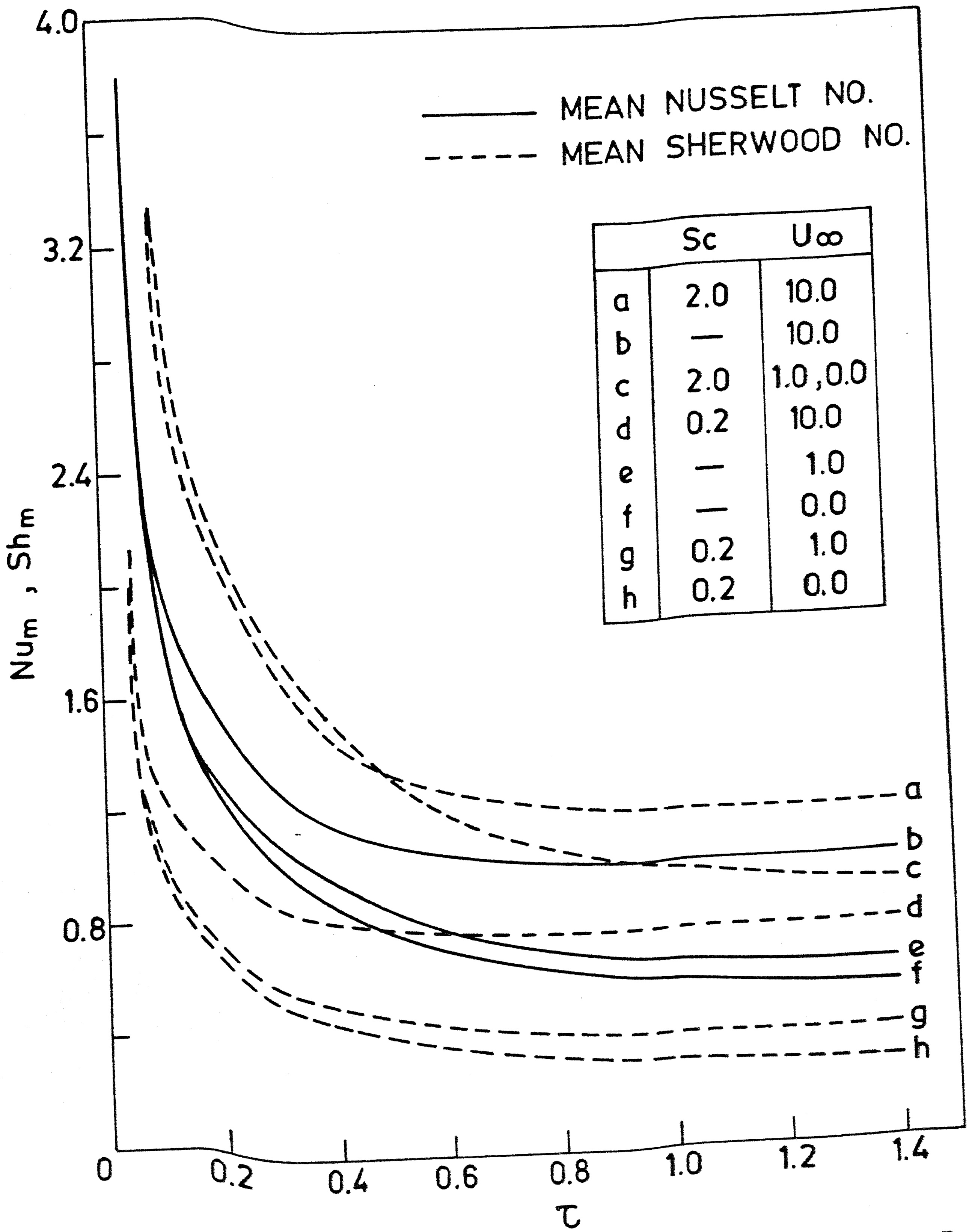

Fig. 4.11 Effect of Sc and U_∞ on the transient mean Nusselt and Sherwood nos. for Pr = 0.7 and N = 0.0

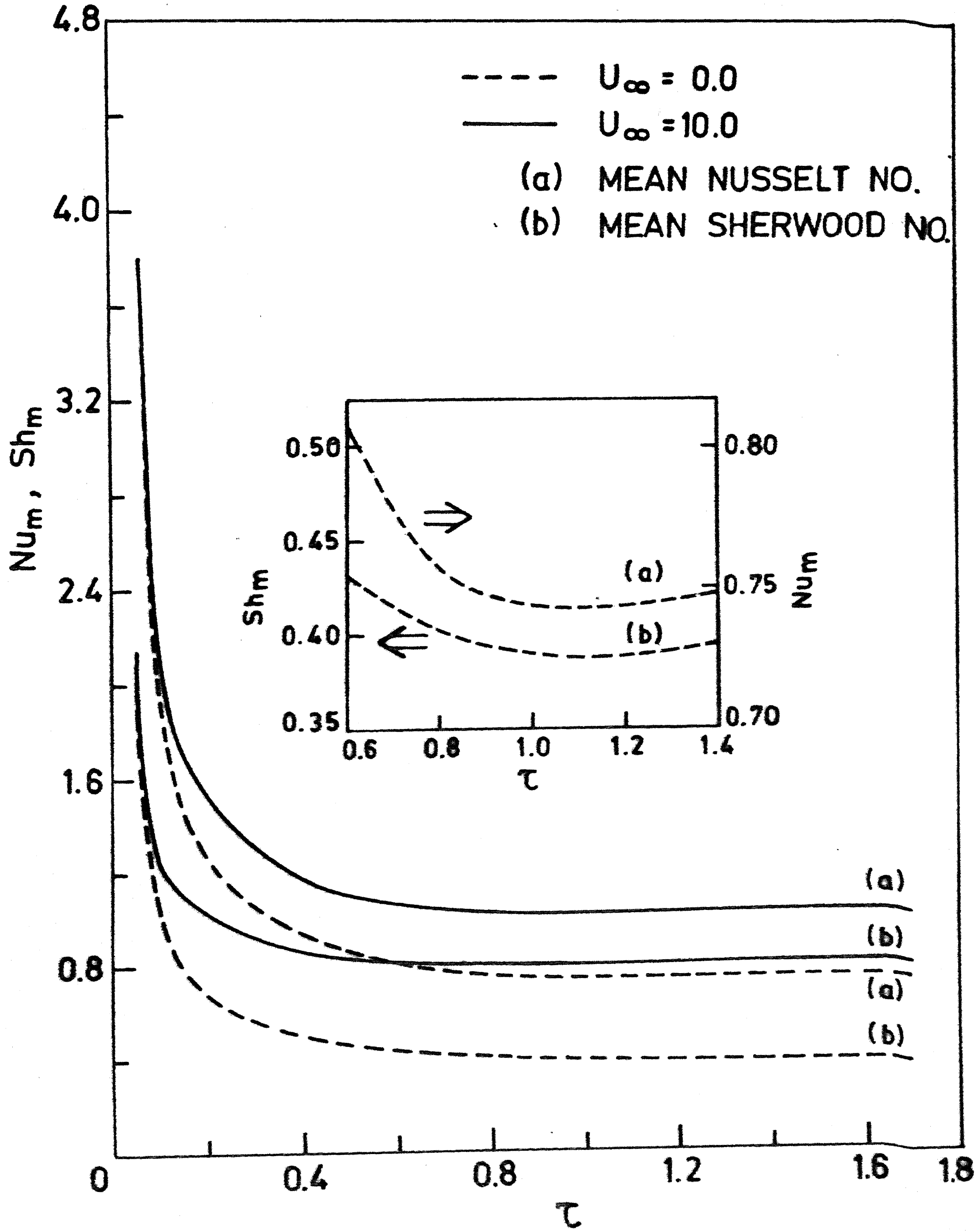

Fig. 4.12 Effect of U_∞ on the transient mean Nusselt and Sherwood nos. for $Pr=0.7$, $Sc=0.2$ and $N=2.0$

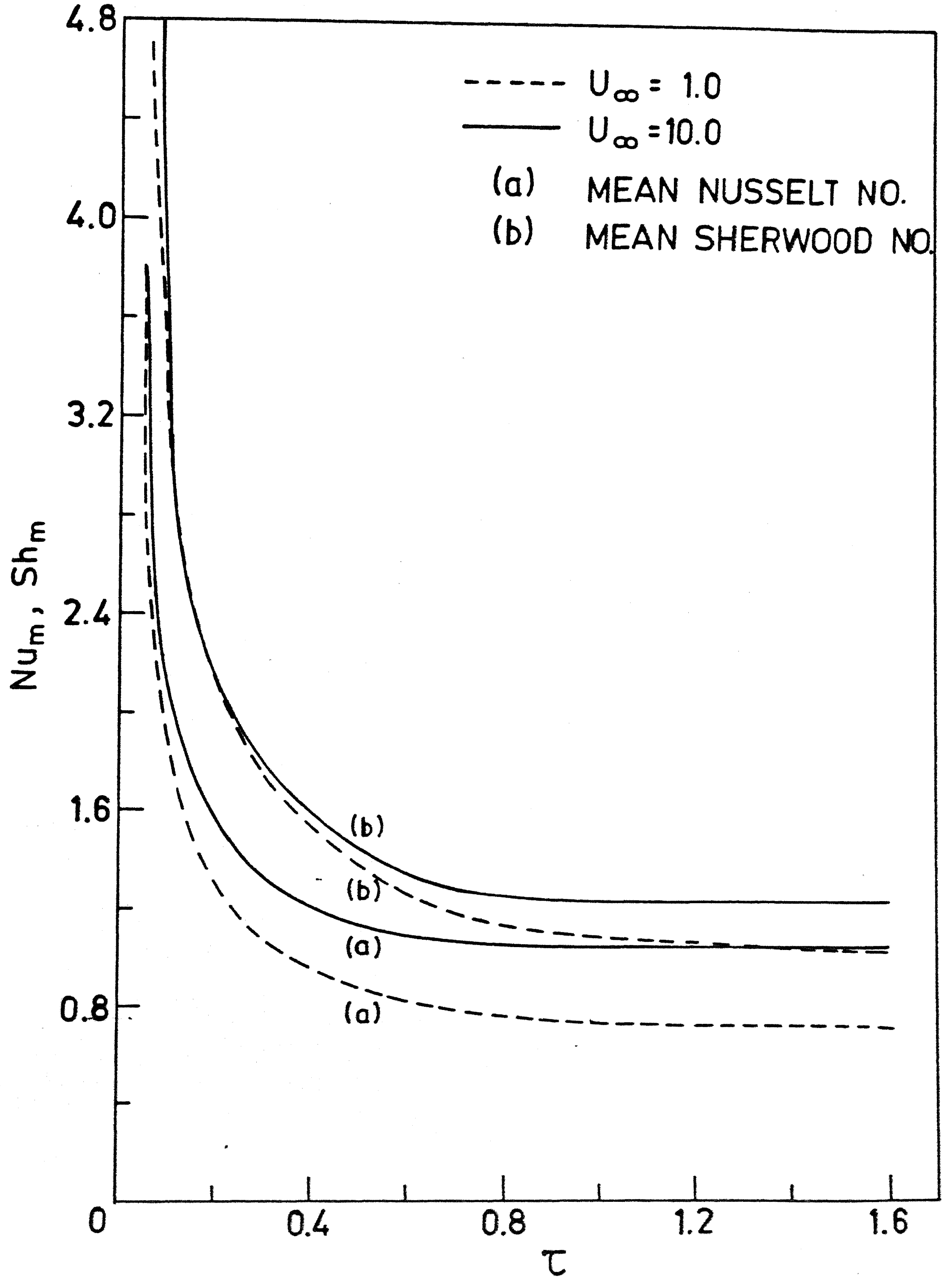

Fig. 4.13 Effect of U_∞ on the transient mean Nusselt and Sherwood nos. for Pr = 0.7, Sc = 2.0, N = 2.0

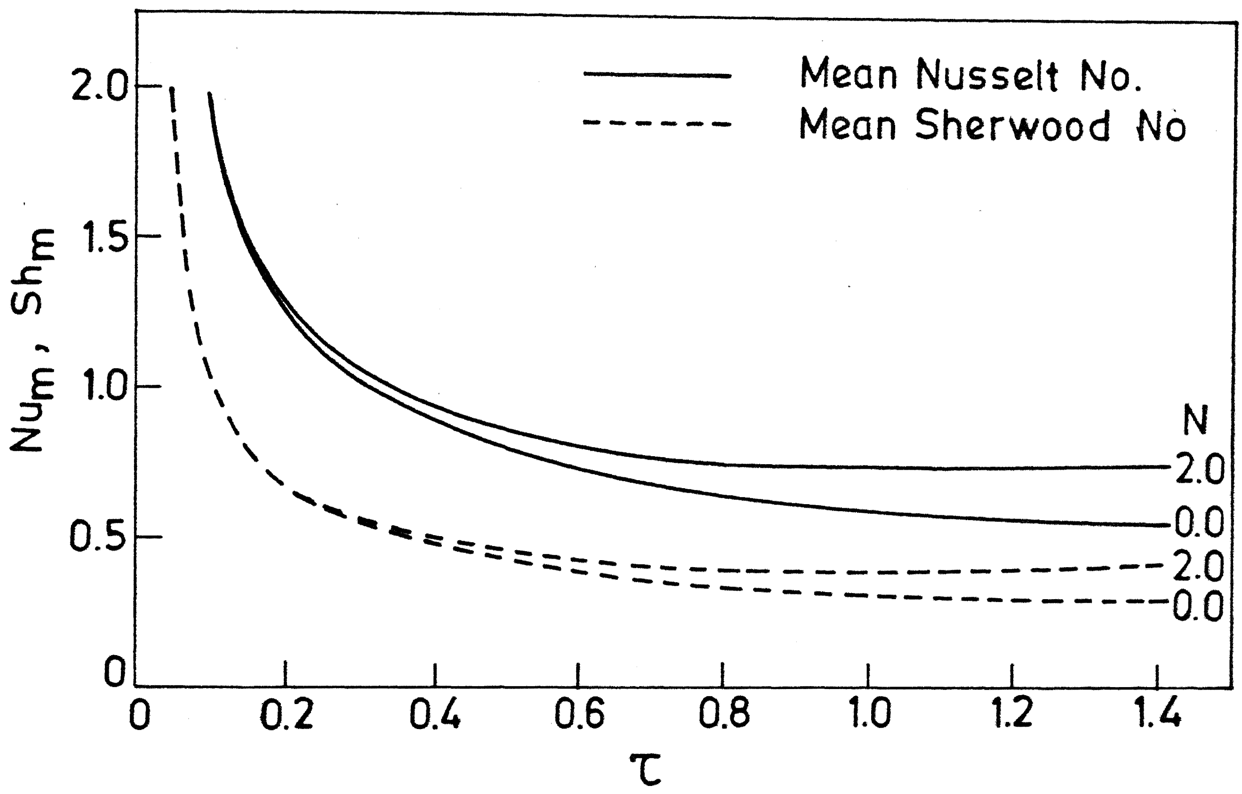

Fig. 4.14 The effect of N on the transient mean Nusselt and Sherwood numbers for $Pr = 0.7, Sc = 0.2, U_\infty = 0.0$

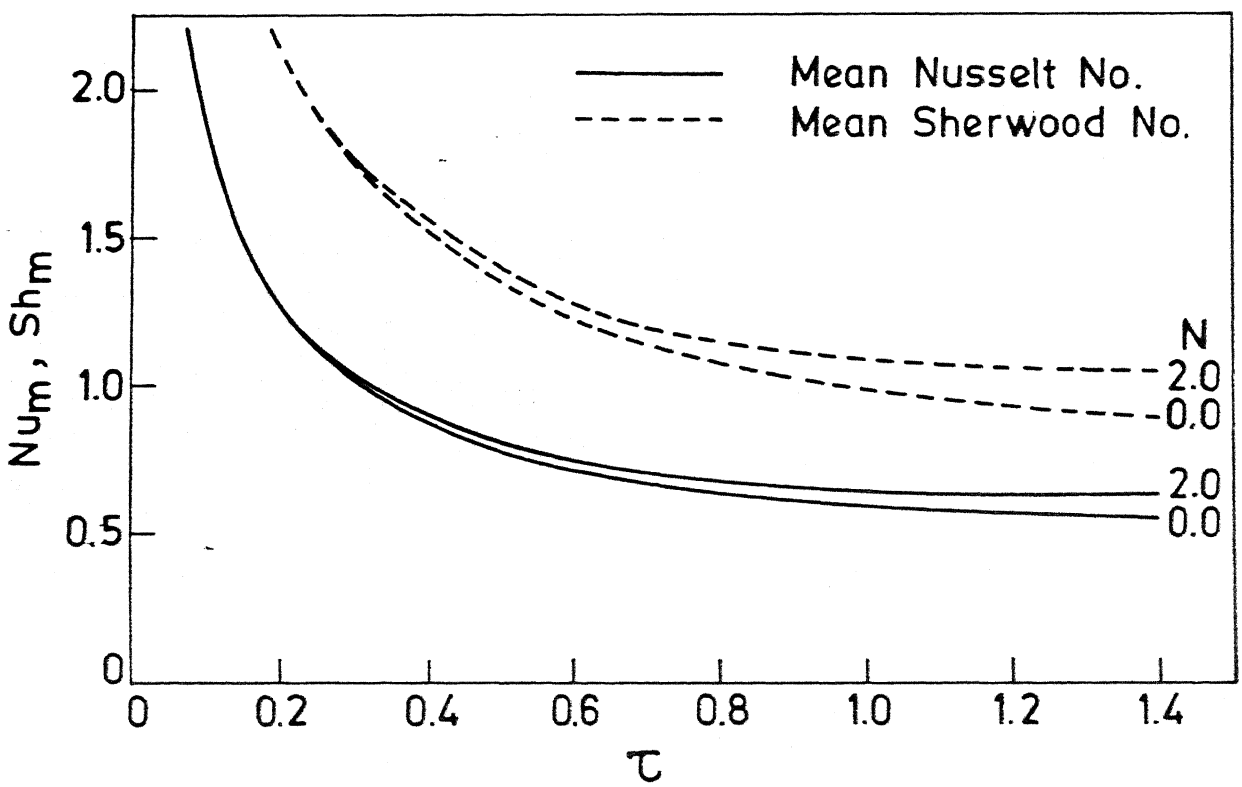

Fig. 4.15 The effect of N on the transient mean Nusselt and Sherwood numbers for $Pr = 0.7, Sc = 2.0, U_\infty = 0.0$

K	Y	U	V	THETA	CONC
1	0.05	0.0438	-0.0007	0.9815	0.9905
2	0.10	0.0851	-0.0020	0.9631	0.9810
3	0.15	0.1241	-0.0039	0.9446	0.9715
4	0.20	0.1607	-0.0065	0.9261	0.9620
5	0.25	0.1950	-0.0098	0.9077	0.9524
6	0.30	0.2271	-0.0137	0.8892	0.9429
7	0.35	0.2570	-0.0182	0.8708	0.9334
8	0.40	0.2847	-0.0231	0.8524	0.9239
9	0.45	0.3104	-0.0292	0.8341	0.9145
10	0.50	0.3342	-0.0356	0.8158	0.9050
11	0.65	0.3936	-0.0602	0.7613	0.8766
12	0.80	0.4374	-0.0899	0.7076	0.8483
13	0.95	0.4671	-0.1242	0.6550	0.8202
14	1.10	0.4846	-0.1627	0.6039	0.7921
15	1.25	0.4915	-0.2049	0.5546	0.7648
16	1.40	0.4897	-0.2501	0.5073	0.7375
17	1.55	0.4806	-0.2977	0.4623	0.7107
18	1.70	0.4657	-0.3471	0.4198	0.6942
19	1.85	0.4464	-0.3976	0.3799	0.6533
20	2.00	0.4238	-0.4487	0.3426	0.6328
21	2.30	0.3722	-0.5496	0.2764	0.5836
22	2.60	0.3182	-0.6458	0.2207	0.5363
23	2.90	0.2663	-0.7346	0.1747	0.4926
24	3.20	0.2189	-0.8144	0.1374	0.4511
25	3.50	0.1773	-0.8847	0.1074	0.4123
26	3.80	0.1418	-0.9452	0.0835	0.3750
27	4.10	0.1121	-0.9965	0.0646	0.3424
28	4.40	0.0877	-1.0393	0.0498	0.3112
29	4.70	0.0680	-1.0745	0.0382	0.2824
30	5.00	0.0523	-1.1031	0.0292	0.2559
31	5.30	0.0399	-1.1261	0.0222	0.2314
32	5.60	0.0302	-1.1443	0.0168	0.2090
33	5.90	0.0227	-1.1567	0.0127	0.1884
34	6.20	0.0169	-1.1698	0.0095	0.1695
35	6.50	0.0125	-1.1784	0.0071	0.1524
36	6.80	0.0092	-1.1850	0.0053	0.1367
37	7.10	0.0067	-1.1900	0.0039	0.1225
38	7.40	0.0049	-1.1937	0.0029	0.1095
39	7.70	0.0035	-1.1965	0.0021	0.0977
40	8.00	0.0025	-1.1986	0.0015	0.0871
41	8.50	0.0014	-1.2006	0.0009	0.0716
42	9.00	0.0008	-1.2019	0.0005	0.0556
43	9.50	0.0004	-1.2026	0.0003	0.0477
44	10.00	0.0002	-1.2030	0.0002	0.0386
45	10.50	0.0001	-1.2032	0.0001	0.0312
46	11.00	0.0001	-1.2033	0.0001	0.0250
47	11.50	0.0000	-1.2034	0.0000	0.0200
48	12.00	0.0000	-1.2034	0.0000	0.0159
49	12.50	0.0000	-1.2034	0.0000	0.0125
50	13.00	0.0000	-1.2035	0.0000	0.0099
51	13.50	0.0000	-1.2035	0.0000	0.0077
52	14.00	0.0000	-1.2035	0.0000	0.0060
53	14.50	0.0000	-1.2035	0.0000	0.0046
54	15.00	0.0000	-1.2035	0.0000	0.0035
55	15.50	0.0000	-1.2035	0.0000	0.0026
56	16.00	0.0000	-1.2035	0.0000	0.0019
57	16.50	0.0000	-1.2035	0.0000	0.0014
58	17.00	0.0000	-1.2035	0.0000	0.0009
59	17.50	0.0000	-1.2035	0.0000	0.0006
60	18.00	0.0000	-1.2035	0.0000	0.0003

MEAN NUSSELT NO= .5445   MEAN SHERWOOD NO= .2764

Time required to reach steady state = 3.2

Table 4.1 : Steady state Velocity ,Temperature & Concentration distributions at X=1.0 for Pr=0.7,Sc=0.2,N=0.0 and U_{∞}=0.0

K	Y	U	V	THETA	CONC
1	0.03	0.0353	-0.0001	0.9873	0.9925
2	0.06	0.0697	-0.0002	0.9745	0.9850
3	0.09	0.1032	-0.0005	0.9618	0.9776
4	0.12	0.1358	-0.0008	0.9490	0.9701
5	0.15	0.1676	-0.0012	0.9363	0.9626
6	0.18	0.1986	-0.0017	0.9236	0.9551
7	0.21	0.2287	-0.0023	0.9109	0.9476
8	0.24	0.2580	-0.0030	0.8982	0.9402
9	0.27	0.2865	-0.0037	0.8855	0.9327
10	0.30	0.3142	-0.0045	0.8728	0.9252
11	0.40	0.4010	-0.0081	0.8306	0.9003
12	0.50	0.4796	-0.0126	0.7887	0.8755
13	0.60	0.5505	-0.0178	0.7472	0.8507
14	0.70	0.6142	-0.0238	0.7061	0.8259
15	0.80	0.6711	-0.0304	0.6657	0.8015
16	0.90	0.7218	-0.0377	0.6260	0.7770
17	1.00	0.7666	-0.0455	0.5871	0.7528
18	1.10	0.8061	-0.0538	0.5492	0.7287
19	1.20	0.8407	-0.0625	0.5123	0.7048
20	1.30	0.8708	-0.0714	0.4767	0.6812
21	1.40	0.8968	-0.0807	0.4423	0.6578
22	1.50	0.9192	-0.0900	0.4092	0.6348
23	1.60	0.9382	-0.0995	0.3776	0.6121
24	1.70	0.9544	-0.1090	0.3475	0.5897
25	1.80	0.9679	-0.1184	0.3189	0.5677
26	1.90	0.9791	-0.1278	0.2918	0.5460
27	2.00	0.9883	-0.1369	0.2663	0.5248
28	2.25	1.0037	-0.1583	0.2094	0.4736
29	2.50	1.0116	-0.1775	0.1619	0.4253
30	2.75	1.0146	-0.1944	0.1231	0.3800
31	3.00	1.0147	-0.2089	0.0921	0.3378
32	3.25	1.0133	-0.2209	0.0678	0.2989
33	3.50	1.0111	-0.2306	0.0491	0.2631
34	3.75	1.0089	-0.2384	0.0350	0.2305
35	4.00	1.0068	-0.2444	0.0246	0.2009
36	4.25	1.0050	-0.2489	0.0169	0.1742
37	4.50	1.0036	-0.2522	0.0114	0.1503
38	5.00	1.0017	-0.2557	0.0052	0.1103
39	5.50	1.0008	-0.2571	0.0022	0.0794
40	6.00	1.0003	-0.2582	0.0009	0.0560
41	6.50	1.0001	-0.2585	0.0004	0.0388
42	7.00	1.0001	-0.2586	0.0001	0.0263
43	7.50	1.0000	-0.2587	0.0001	0.0175
44	8.00	1.0000	-0.2587	0.0000	0.0115
45	8.50	1.0000	-0.2587	0.0000	0.0073
46	9.00	1.0000	-0.2587	0.0000	0.0046
47	9.50	1.0000	-0.2587	0.0000	0.0028
48	10.00	1.0000	-0.2587	0.0000	0.0017
49	10.50	1.0000	-0.2587	0.0000	0.0009
50	11.00	1.0000	-0.2587	0.0000	0.0004

MEAN NUSSELT NO= .6298 MEAN SHERWOOD NO= .4013

Time required to reach steady state = 2.0

Table 4.2: Steady state Velocity, Temperature, Concentration distributions at Y=1.0 and U_∞=1.0 for Pr=0.7, Sc=0.2, N=0.0

K	Y	U	V	THETA	CONC
1	0.01	0.0840	0.0002	0.9926	0.9947
2	0.02	0.1678	0.0007	0.9851	0.9894
3	0.03	0.2516	0.0015	0.9777	0.9840
4	0.04	0.3353	0.0025	0.9702	0.9787
5	0.05	0.4189	0.0037	0.9628	0.9734
6	0.06	0.5023	0.0052	0.9553	0.9681
7	0.07	0.5857	0.0070	0.9479	0.9627
8	0.08	0.6690	0.0090	0.9404	0.9574
9	0.09	0.7521	0.0112	0.9330	0.9521
10	0.10	0.8352	0.0137	0.9255	0.9468
11	0.14	1.1664	0.0276	0.8958	0.9255
12	0.18	1.4959	0.0454	0.8660	0.9042
13	0.22	1.8234	0.0672	0.8363	0.8829
14	0.26	2.1488	0.0929	0.8067	0.8616
15	0.30	2.4717	0.1223	0.7772	0.8404
16	0.34	2.7920	0.1555	0.7477	0.8192
17	0.38	3.1092	0.1924	0.7185	0.7981
18	0.42	3.4230	0.2327	0.6894	0.7769
19	0.46	3.7329	0.2764	0.6605	0.7559
20	0.50	4.0384	0.3232	0.6319	0.7349
21	0.60	4.7811	0.4578	0.5617	0.6829
22	0.70	5.4855	0.6056	0.4941	0.6316
23	0.80	6.1437	0.7614	0.4299	0.5813
24	0.90	6.7484	0.9196	0.3696	0.5322
25	1.00	7.2940	1.0747	0.3140	0.4847
26	1.10	7.7771	1.2215	0.2635	0.4390
27	1.20	8.1971	1.3561	0.2184	0.3953
28	1.30	8.5560	1.4758	0.1787	0.3539
29	1.40	8.8583	1.5795	0.1444	0.3150
30	1.50	9.1101	1.6673	0.1150	0.2788
31	2.00	9.7132	1.8227	0.0384	0.1394
32	2.50	9.9081	1.8727	0.0124	0.0622
33	3.00	9.9703	1.8885	0.0040	0.0251
34	3.50	9.9903	1.8936	0.0013	0.0097
35	4.00	9.9968	1.8953	0.0004	0.0035
36	4.50	9.9989	1.8959	0.0001	0.0012
37	5.00	9.9997	1.8960	0.0000	0.0004
38	5.50	9.9999	1.8961	0.0000	0.0001
39	6.00	10.0000	1.8961	0.0000	0.0000

MEAN NUSSELT NO=1.0192 MEAN SHERWOOD NO= .7820

Time required to reach steady state = 2.0

Table 4.3: Steady state Velocity, Temperature & Concentration distributions at X=1.0 for Pr=0.7, Sc=0.2, N=0.0 and U_∞=10.0

K	Y	U	V	THET	CON
1	0.05	0.0180	0.0000	0.8295	0.9047
2	0.10	0.0304	0.0000	0.6879	0.8185
3	0.15	0.0384	0.0000	0.5705	0.7405
4	0.20	0.0433	0.0000	0.4730	0.6699
5	0.25	0.0457	0.0000	0.3921	0.6060
6	0.30	0.0465	0.0000	0.3249	0.5481
7	0.35	0.0460	0.0000	0.2691	0.4957
8	0.40	0.0447	0.0000	0.2227	0.4483
9	0.45	0.0428	0.0000	0.1841	0.4053
10	0.50	0.0405	0.0000	0.1519	0.3664
11	0.65	0.0327	0.0000	0.0873	0.2718
12	0.80	0.0254	0.0000	0.0501	0.2015
13	0.95	0.0192	0.0000	0.0288	0.1495
14	1.10	0.0144	0.0000	0.0166	0.1108
15	1.25	0.0107	0.0000	0.0095	0.0822
16	1.40	0.0079	0.0000	0.0055	0.0609
17	1.55	0.0059	0.0000	0.0031	0.0451
18	1.70	0.0043	0.0000	0.0018	0.0334
19	1.85	0.0032	0.0000	0.0010	0.0247
20	2.00	0.0023	0.0000	0.0006	0.0182
21	2.30	0.0013	0.0000	0.0002	0.0101
22	2.60	0.0007	0.0000	0.0001	0.0056
23	2.90	0.0004	0.0000	0.0000	0.0031
24	3.20	0.0002	0.0000	0.0000	0.0017
25	3.50	0.0001	0.0000	0.0000	0.0009
26	3.80	0.0001	0.0000	0.0000	0.0005
27	4.10	0.0000	0.0000	0.0000	0.0003
28	4.40	0.0000	0.0000	0.0000	0.0002
29	4.70	0.0000	0.0000	0.0000	0.0001
30	5.00	0.0000	0.0000	0.0000	0.0000

MEAN NUSSELT NO=3.6818 MEAN SHERWOOD NO=1.9854

Table 4.4 a : Tau = 0.05 : Velocity, Temperature and Concentration distribution at X = 1.0 for Pr = 0.7, Sc = 0.2, N =2 and U_∞ = 0.0 .

K	Y	U	V	THET	COM
1	0.05	0.0548	0.0000	0.9539	0.9751
2	0.10	0.1025	0.0000	0.9081	0.9508
3	0.15	0.1438	0.0000	0.8625	0.9263
4	0.20	0.1790	0.0000	0.8175	0.9018
5	0.25	0.2088	0.0000	0.7732	0.8775
6	0.30	0.2335	0.0000	0.7298	0.8533
7	0.35	0.2536	0.0000	0.6973	0.8292
8	0.40	0.2695	0.0000	0.6460	0.8051
9	0.45	0.2817	0.0000	0.6058	0.7817
10	0.50	0.2906	0.0000	0.5671	0.7583
11	0.65	0.2995	0.0000	0.4592	0.6896
12	0.80	0.2905	0.0000	0.3656	0.6237
13	0.95	0.2705	0.0000	0.2863	0.5611
14	1.10	0.2447	0.0000	0.2209	0.5022
15	1.25	0.2167	0.0000	0.1680	0.4472
16	1.40	0.1887	0.0000	0.1261	0.3962
17	1.55	0.1623	0.0000	0.0935	0.3494
18	1.70	0.1382	0.0000	0.0684	0.3066
19	1.85	0.1167	0.0000	0.0495	0.2679
20	2.00	0.0980	0.0000	0.0353	0.2331
21	2.30	0.0684	0.0000	0.0179	0.1744
22	2.60	0.0472	0.0000-	0.0088	0.1286
23	2.90	0.0322	0.0000	0.0043	0.0935
24	3.20	0.0219	0.0000	0.0020	0.0671
25	3.50	0.0148	0.0000	0.0009	0.0475
26	3.80	0.0099	0.0000	0.0004	0.0334
27	4.10	0.0066	0.0000	0.0002	0.0232
28	4.40	0.0044	0.0000	0.0001	0.0160
29	4.70	0.0029	0.0000	0.0000	0.0109
30	5.00	0.0019	0.0000	0.0000	0.0074
31	5.30	0.0012	0.0000	0.0000	0.0050
32	5.60	0.0008	0.0000	0.0000	0.0033
33	5.90	0.0005	0.0000	0.0000	0.0022
34	6.20	0.0003	0.0000	0.0000	0.0015
35	6.50	0.0002	0.0000	0.0000	0.0010
36	6.80	0.0001	0.0000	0.0000	0.0006
37	7.10	0.0001	0.0000	0.0000	0.0004
38	7.40	0.0001	0.0000	0.0000	0.0003
39	7.70	0.0000	0.0000	0.0000	0.0002
40	8.00	0.0000	0.0000	0.0000	0.0001
41	8.50	0.0000	0.0000	0.0000	0.0000
42	9.00	0.0000	0.0000	0.0000	0.0000
43	9.50	0.0000	0.0000	0.0000	0.0000
44	10.00	0.0000	0.0000	0.0000	0.0000
45	10.50	0.0000	0.0000	0.0000	0.0000
46	11.00	0.0000	0.0000	0.0000	0.0000
47	11.50	0.0000	0.0000	0.0000	0.0000
48	12.00	0.0000	0.0000	0.0000	0.0000
49	12.50	0.0000	0.0000	0.0000	0.0000
50	13.00	0.0000	0.0000	0.0000	0.0000
51	13.50	0.0000	0.0000	0.0000	0.0000
52	14.00	0.0000	0.0000	0.0000	0.0000
53	14.50	0.0000	0.0000	0.0000	0.0000
54	15.00	0.0000	0.0000	0.0000	0.0000
55	15.50	0.0000	0.0000	0.0000	0.0000
56	16.00	0.0000	0.0000	0.0000	0.0000
57	16.50	0.0000	0.0000	0.0000	0.0000
58	17.00	0.0000	0.0000	0.0000	0.0000

MEAN NUSSELT NO=1.0579 MEAN SHERWOOD NO= .5650

Table 4.4 b : Tau = 0.3 : Velocity, Temperature and Concentration distribution at X = 1.0 for Pr = 0.7, Sc = 0.2, N = 2 and U_∞ = 0.0 .

K	Y	U	V	THET	CON
1	0.05	0.0641	0.0000	0.9608	0.9790
2	0.10	0.1212	0.0000	0.9217	0.9581
3	0.15	0.1716	0.0000	0.8829	0.9372
4	0.20	0.2158	0.0000	0.8443	0.9164
5	0.25	0.2542	0.0000	0.8062	0.8956
6	0.30	0.2872	0.0000	0.7685	0.8748
7	0.35	0.3152	0.0000	0.7315	0.8542
8	0.40	0.3386	0.0000	0.6951	0.8337
9	0.45	0.3578	0.0000	0.6596	0.8133
10	0.50	0.3731	0.0000	0.6248	0.7931
11	0.65	0.3978	0.0000	0.5260	0.7333
12	0.80	0.3994	0.0000	0.4367	0.6752
13	0.95	0.3850	0.0000	0.3578	0.6193
14	1.10	0.3603	0.0000	0.2893	0.5656
15	1.25	0.3297	0.0000	0.2310	0.5146
16	1.40	0.2965	0.0000	0.1822	0.4663
17	1.55	0.2631	0.0000	0.1420	0.4208
18	1.70	0.2309	0.0000	0.1095	0.3784
19	1.85	0.2009	0.0000	0.0831	0.3389
20	2.00	0.1735	0.0000	0.0628	0.3025
21	2.30	0.1275	0.0000	0.0351	0.2384
22	2.60	0.0924	0.0000	0.0191	0.1853
23	2.90	0.0662	0.0000	0.0101	0.1422
24	3.20	0.0471	0.0000	0.0052	0.1078
25	3.50	0.0333	0.0000	0.0026	0.0808
26	3.80	0.0234	0.0000	0.0013	0.0599
27	4.10	0.0164	0.0000	0.0006	0.0439
28	4.40	0.0114	0.0000	0.0003	0.0319
29	4.70	0.0079	0.0000	0.0001	0.0230
30	5.00	0.0054	0.0000	0.0001	0.0164
31	5.30	0.0037	0.0000	0.0000	0.0116
32	5.60	0.0025	0.0000	0.0000	0.0081
33	5.90	0.0017	0.0000	0.0000	0.0057
34	6.20	0.0011	0.0000	0.0000	0.0039
35	6.50	0.0008	0.0000	0.0000	0.0027
36	6.80	0.0005	0.0000	0.0000	0.0019
37	7.10	0.0003	0.0000	0.0000	0.0013
38	7.40	0.0002	0.0000	0.0000	0.0008
39	7.70	0.0001	0.0000	0.0000	0.0006
40	8.00	0.0001	0.0000	0.0000	0.0004
41	8.50	0.0000	0.0000	0.0000	0.0002
42	9.00	0.0000	0.0000	0.0000	0.0001
43	9.50	0.0000	0.0000	0.0000	0.0000
44	10.00	0.0000	0.0000	0.0000	0.0000
45	10.50	0.0000	0.0000	0.0000	0.0000
46	11.00	0.0000	0.0000	0.0000	0.0000
47	11.50	0.0000	0.0000	0.0000	0.0000
48	12.00	0.0000	0.0000	0.0000	0.0000
49	12.50	0.0000	0.0000	0.0000	0.0000
50	13.00	0.0000	0.0000	0.0000	0.0000
51	13.50	0.0000	0.0000	0.0000	0.0000
52	14.00	0.0000	0.0000	0.0000	0.0000
53	14.50	0.0000	0.0000	0.0000	0.0000
54	15.00	0.0000	0.0000	0.0000	0.0000
55	15.50	0.0000	0.0000	0.0000	0.0000
56	16.00	0.0000	0.0000	0.0000	0.0000
57	16.50	0.0000	0.0000	0.0000	0.0000
58	17.00	0.0000	0.0000	0.0000	0.0000

MEAN NUSSELT NO= .9550 MEAN SHERWOOD NO= .5103

Table 4.4 c : Tau = 0.4 : Velocity,Temperature and Concentration distribution at X = 1.0 for Pr = 0.7,Sc = 0.2,M =2 and U_∞ = 0.0 .

K	Y	U	V	THET	CON
1	0.05	0.0912	-0.0002	0.9717	0.9848
2	0.10	0.1753	-0.0005	0.9434	0.9697
3	0.15	0.2524	-0.0009	0.9152	0.9545
4	0.20	0.3230	-0.0015	0.8871	0.9394
5	0.25	0.3872	-0.0023	0.8592	0.9243
6	0.30	0.4454	-0.0033	0.8314	0.9092
7	0.35	0.4980	-0.0044	0.8039	0.8942
8	0.40	0.5451	-0.0057	0.7766	0.8792
9	0.45	0.5870	-0.0071	0.7496	0.8642
10	0.50	0.6241	-0.0088	0.7230	0.8493
11	0.65	0.7079	-0.0155	0.6451	0.8049
12	0.80	0.7576	-0.0239	0.5711	0.7612
13	0.95	0.7796	-0.0341	0.5018	0.7183
14	1.10	0.7799	-0.0460	0.4376	0.6763
15	1.25	0.7635	-0.0594	0.3789	0.6355
16	1.40	0.7349	-0.0741	0.3256	0.5958
17	1.55	0.6978	-0.0900	0.2780	0.5574
18	1.70	0.6553	-0.1066	0.2357	0.5205
19	1.85	0.6097	-0.1237	0.1985	0.4849
20	2.00	0.5630	-0.1410	0.1662	0.4509
21	2.30	0.4707	-0.1748	0.1149	0.3876
22	2.60	0.3866	-0.2063	0.0778	0.3306
23	2.90	0.3135	-0.2346	0.0518	0.2799
24	3.20	0.2519	-0.2593	0.0340	0.2354
25	3.50	0.2011	-0.2805	0.0220	0.1966
26	3.80	0.1597	-0.2984	0.0141	0.1632
27	4.10	0.1264	-0.3133	0.0089	0.1346
28	4.40	0.0997	-0.3255	0.0056	0.1105
29	4.70	0.0784	-0.3358	0.0035	0.0901
30	5.00	0.0615	-0.3441	0.0021	0.0732
31	5.30	0.0482	-0.3509	0.0013	0.0591
32	5.60	0.0377	-0.3564	0.0008	0.0475
33	5.90	0.0294	-0.3608	0.0005	0.0381
34	6.20	0.0229	-0.3644	0.0003	0.0304
35	6.50	0.0178	-0.3673	0.0002	0.0241
36	6.80	0.0138	-0.3696	0.0001	0.0191
37	7.10	0.0106	-0.3715	0.0001	0.0151
38	7.40	0.0082	-0.3730	0.0000	0.0118
39	7.70	0.0063	-0.3742	0.0000	0.0093
40	8.00	0.0049	-0.3751	0.0000	0.0072
41	8.50	0.0031	-0.3762	0.0000	0.0048
42	9.00	0.0020	-0.3769	0.0000	0.0031
43	9.50	0.0013	-0.3774	0.0000	0.0021
44	10.00	0.0008	-0.3777	0.0000	0.0013
45	10.50	0.0005	-0.3780	0.0000	0.0009
46	11.00	0.0003	-0.3781	0.0000	0.0006
47	11.50	0.0002	-0.3782	0.0000	0.0004
48	12.00	0.0001	-0.3783	0.0000	0.0002
49	12.50	0.0001	-0.3783	0.0000	0.0001
50	13.00	0.0001	-0.3783	0.0000	0.0001
51	13.50	0.0000	-0.3784	0.0000	0.0001
52	14.00	0.0000	-0.3784	0.0000	0.0000
53	14.50	0.0000	-0.3784	0.0000	0.0000
54	15.00	0.0000	-0.3784	0.0000	0.0000
55	15.50	0.0000	-0.3784	0.0000	0.0000
56	16.00	0.0000	-0.3784	0.0000	0.0000
57	16.50	0.0000	-0.3784	0.0000	0.0000
58	17.00	0.0000	-0.3784	0.0000	0.0000

MEAN NUSSELT NO= .7573 MEAN SHERWOOD NO= .4039

Table 4.4 d : Tau = 0.8 : Velocity,Temperature and Concentration distribution at $X = 1.0$ for $Pr = 0.7$, $Sc = 0.2$, $N = 2$ and $U_\infty = 0.0$.

K	Y	U	V	THET	CON
1	0.05	0.1072	-0.0008	0.9745	0.9853
2	0.10	0.2072	-0.0025	0.9490	0.9726
3	0.15	0.3000	-0.0050	0.9236	0.9590
4	0.20	0.3861	-0.0084	0.8981	0.9453
5	0.25	0.4655	-0.0125	0.8728	0.9316
6	0.30	0.5385	-0.0175	0.8475	0.9180
7	0.35	0.6054	-0.0236	0.8223	0.9044
8	0.40	0.6664	-0.0304	0.7972	0.8908
9	0.45	0.7217	-0.0381	0.7722	0.8772
10	0.50	0.7716	-0.0467	0.7475	0.8636
11	0.65	0.8899	-0.0804	0.6743	0.8231
12	0.80	0.9678	-0.1220	0.6037	0.7830
13	0.95	1.0116	-0.1711	0.5363	0.7434
14	1.10	1.0272	-0.2271	0.4728	0.7045
15	1.25	1.0202	-0.2890	0.4138	0.6664
16	1.40	0.9959	-0.3558	0.3595	0.6293
17	1.55	0.9586	-0.4263	0.3101	0.5932
18	1.70	0.9122	-0.4992	0.2658	0.5582
19	1.85	0.8601	-0.5733	0.2264	0.5244
20	2.00	0.8047	-0.6476	0.1918	0.4919
21	2.30	0.6906	-0.7909	0.1357	0.4307
22	2.60	0.5824	-0.9235	0.0943	0.3750
23	2.90	0.4854	-1.0427	0.0646	0.3248
24	3.20	0.4012	-1.1473	0.0437	0.2798
25	3.50	0.3298	-1.2379	0.0292	0.2393
26	3.80	0.2699	-1.3153	0.0193	0.2048
27	4.10	0.2202	-1.3808	0.0126	0.1740
28	4.40	0.1791	-1.4361	0.0082	0.1472
29	4.70	0.1454	-1.4824	0.0053	0.1240
30	5.00	0.1178	-1.5211	0.0034	0.1040
31	5.30	0.0952	-1.5533	0.0021	0.0869
32	5.60	0.0768	-1.5801	0.0014	0.0723
33	5.90	0.0619	-1.6022	0.0008	0.0600
34	6.20	0.0497	-1.6206	0.0005	0.0495
35	6.50	0.0398	-1.6357	0.0003	0.0408
36	6.80	0.0319	-1.6481	0.0002	0.0334
37	7.10	0.0254	-1.6583	0.0001	0.0273
38	7.40	0.0202	-1.6667	0.0001	0.0223
39	7.70	0.0161	-1.6735	0.0000	0.0181
40	8.00	0.0127	-1.6796	0.0000	0.0146
41	8.50	0.0086	-1.6856	0.0000	0.0102
42	9.00	0.0058	-1.6902	0.0000	0.0071
43	9.50	0.0039	-1.6931	0.0000	0.0049
44	10.00	0.0026	-1.6956	0.0000	0.0034
45	10.50	0.0017	-1.6972	0.0000	0.0023
46	11.00	0.0012	-1.6983	0.0000	0.0016
47	11.50	0.0008	-1.6990	0.0000	0.0011
48	12.00	0.0005	-1.6995	0.0000	0.0007
49	12.50	0.0003	-1.6998	0.0000	0.0005
50	13.00	0.0002	-1.7000	0.0000	0.0003
51	13.50	0.0001	-1.7002	0.0000	0.0002
52	14.00	0.0001	-1.7003	0.0000	0.0001
53	14.50	0.0001	-1.7004	0.0000	0.0001
54	15.00	0.0000	-1.7004	0.0000	0.0001
55	15.50	0.0000	-1.7004	0.0000	0.0000
56	16.00	0.0000	-1.7005	0.0000	0.0000
57	16.50	0.0000	-1.7005	0.0000	0.0000
58	17.00	0.0000	-1.7005	0.0000	0.0000

MEAN NUSSELT NU= .7431 MEAN SHERWOOD NO= .3934

Table 4.4 e : Tau = 1.0 : Velocity,Temperature and Concentration distribution at X = 1.0 for Pr = 0.7,Sc = 0.2,M =2 and U_∞ = 0.0 .

K	Y	U	V	THETA	CONC
1	0.05	0.1146	-0.0016	0.9730	0.9860
2	0.10	0.2219	-0.0049	0.9461	0.9719
3	0.15	0.3220	-0.0097	0.9192	0.9579
4	0.20	0.4150	-0.0162	0.8923	0.9439
5	0.25	0.5012	-0.0242	0.8654	0.9298
6	0.30	0.5807	-0.0338	0.8386	0.9158
7	0.35	0.6537	-0.0450	0.8118	0.9018
8	0.40	0.7205	-0.0577	0.7852	0.8878
9	0.45	0.7812	-0.0720	0.7587	0.8739
10	0.50	0.8360	-0.0877	0.7324	0.8599
11	0.65	0.9668	-0.1478	0.6546	0.8183
12	0.80	1.0533	-0.2195	0.5795	0.7770
13	0.95	1.1018	-0.3014	0.5080	0.7364
14	1.10	1.1191	-0.3918	0.4409	0.6961
15	1.25	1.1114	-0.4888	0.3790	0.6571
16	1.40	1.0844	-0.5906	0.3225	0.6194
17	1.55	1.0433	-0.6954	0.2719	0.5826
18	1.70	0.9927	-0.8014	0.2272	0.5470
19	1.85	0.9361	-0.9072	0.1881	0.5129
20	2.00	0.8765	-1.0113	0.1545	0.4801
21	2.30	0.7552	-1.2072	0.1022	0.4190
22	2.60	0.6419	-1.3861	0.0660	0.3639
23	2.90	0.5415	-1.5462	0.0417	0.3148
24	3.20	0.4549	-1.6877	0.0259	0.2714
25	3.50	0.3814	-1.8119	0.0159	0.2332
26	3.80	0.3195	-1.9201	0.0096	0.1999
27	4.10	0.2675	-2.0143	0.0058	0.1709
28	4.40	0.2239	-2.0960	0.0035	0.1458
29	4.70	0.1872	-2.1668	0.0021	0.1242
30	5.00	0.1565	-2.2281	0.0012	0.1055
31	5.30	0.1307	-2.2810	0.0007	0.0895
32	5.60	0.1090	-2.3265	0.0004	0.0759
33	5.90	0.0909	-2.3657	0.0003	0.0643
34	6.20	0.0756	-2.3994	0.0002	0.0543
35	6.50	0.0629	-2.4282	0.0001	0.0458
36	6.80	0.0522	-2.4528	0.0001	0.0386
37	7.10	0.0432	-2.4738	0.0000	0.0325
38	7.40	0.0359	-2.4917	0.0000	0.0273
39	7.70	0.0296	-2.5069	0.0000	0.0229
40	8.00	0.0244	-2.5198	0.0000	0.0192
41	8.50	0.0177	-2.5360	0.0000	0.0143
42	9.00	0.0128	-2.5483	0.0000	0.0106
43	9.50	0.0092	-2.5574	0.0000	0.0078
44	10.00	0.0066	-2.5642	0.0000	0.0057
45	10.50	0.0047	-2.5693	0.0000	0.0042
46	11.00	0.0034	-2.5730	0.0000	0.0031
47	11.50	0.0024	-2.5757	0.0000	0.0022
48	12.00	0.0017	-2.5776	0.0000	0.0015
49	12.50	0.0012	-2.5790	0.0000	0.0012
50	13.00	0.0008	-2.5800	0.0000	0.0008
51	13.50	0.0006	-2.5807	0.0000	0.0006
52	14.00	0.0004	-2.5812	0.0000	0.0004
53	14.50	0.0002	-2.5816	0.0000	0.0003
54	15.00	0.0002	-2.5818	0.0000	0.0002
55	15.50	0.0001	-2.5819	0.0000	0.0001
56	16.00	0.0001	-2.5820	0.0000	0.0001
57	16.50	0.0000	-2.5820	0.0000	0.0000

MEAN NUSSELT NO= .7589 MEAN SHERWOOD NO= .3973

Time required to reach steady state = 2.8

Table 4.4 f: Steady state Velocity, Temperature & Concentration distributions at X=1.0 for Pr=0.7, Sc=0.2, M=2.0 and U_∞ =0.0

K	Y	U	V	THET	CON
1	0.03	0.0674	-0.0000	0.9765	0.9874
2	0.06	0.1322	-0.0000	0.9530	0.9748
3	0.09	0.1944	-0.0000	0.9295	0.9622
4	0.12	0.2540	-0.0001	0.9062	0.9497
5	0.15	0.3111	-0.0001	0.8829	0.9371
6	0.18	0.3659	-0.0001	0.8597	0.9245
7	0.21	0.4182	-0.0002	0.8366	0.9121
8	0.24	0.4682	-0.0002	0.8138	0.8996
9	0.27	0.5159	-0.0002	0.7910	0.8871
10	0.30	0.5615	-0.0003	0.7685	0.8747
11	0.40	0.6976	-0.0006	0.6950	0.8334
12	0.50	0.8118	-0.0009	0.6245	0.7927
13	0.60	0.9061	-0.0014	0.5577	0.7526
14	0.70	0.9826	-0.0020	0.4949	0.7133
15	0.80	1.0433	-0.0027	0.4364	0.6747
16	0.90	1.0902	-0.0035	0.3825	0.6371
17	1.00	1.1252	-0.0045	0.3333	0.6005
18	1.10	1.1501	-0.0056	0.2888	0.5649
19	1.20	1.1665	-0.0069	0.2487	0.5305
20	1.30	1.1761	-0.0083	0.2130	0.4973
21	1.40	1.1801	-0.0097	0.1815	0.4654
22	1.50	1.1796	-0.0113	0.1538	0.4347
23	1.60	1.1759	-0.0130	0.1296	0.4051
24	1.70	1.1696	-0.0147	0.1087	0.3774
25	1.80	1.1616	-0.0164	0.0907	0.3507
26	1.90	1.1525	-0.0182	0.0753	0.3254
27	2.00	1.1427	-0.0200	0.0621	0.3013
28	2.25	1.1169	-0.0244	0.0381	0.2471
29	2.50	1.0930	-0.0284	0.0229	0.2006
30	2.75	1.0724	-0.0320	0.0134	0.1614
31	3.00	1.0556	-0.0351	0.0077	0.1286
32	3.25	1.0421	-0.0378	0.0044	0.1016
33	3.50	1.0317	-0.0400	0.0024	0.0796
34	3.75	1.0236	-0.0418	0.0013	0.0619
35	4.00	1.0175	-0.0433	0.0007	0.0477
36	4.25	1.0128	-0.0445	0.0004	0.0365
37	4.50	1.0093	-0.0454	0.0002	0.0275
38	5.00	1.0050	-0.0467	0.0001	0.0158
39	5.50	1.0026	-0.0474	0.0000	0.0088
40	6.00	1.0014	-0.0479	0.0000	0.0048
41	6.50	1.0007	-0.0482	0.0000	0.0026
42	7.00	1.0003	-0.0483	0.0000	0.0014
43	7.50	1.0002	-0.0484	0.0000	0.0007
44	8.00	1.0001	-0.0485	0.0000	0.0004
45	8.50	1.0000	-0.0485	0.0000	0.0002
46	9.00	1.0000	-0.0485	0.0000	0.0001
47	9.50	1.0000	-0.0485	0.0000	0.0000

MEAN NUSSELT NO=1.0057 MEAN SHERWOOD NO= .5775

Table 1.5 a : Tau = 0.4 : Velocity, Temperature and Concentration distribution at X = 1.0 for Pr = 0.7, Sc = 0.2, [illegible] =2 and U_{∞} = 1.0 .

K	Y	U	V	THET	CON
1	0.03	0.0736	-0.0002	0.9821	0.9902
2	0.06	0.1446	-0.0005	0.9642	0.9803
3	0.09	0.2129	-0.0011	0.9463	0.9705
4	0.12	0.2787	-0.0018	0.9284	0.9606
5	0.15	0.3420	-0.0026	0.9106	0.9508
6	0.18	0.4028	-0.0039	0.8928	0.9409
7	0.21	0.4612	-0.0052	0.8750	0.9311
8	0.24	0.5173	-0.0066	0.8573	0.9213
9	0.27	0.5710	-0.0083	0.8396	0.9115
10	0.30	0.6225	-0.0102	0.8220	0.9017
11	0.40	0.7780	-0.0185	0.7639	0.8691
12	0.50	0.9108	-0.0291	0.7069	0.8366
13	0.60	1.0226	-0.0418	0.6513	0.8044
14	0.70	1.1153	-0.0568	0.5974	0.7725
15	0.80	1.1908	-0.0739	0.5455	0.7410
16	0.90	1.2508	-0.0932	0.4958	0.7098
17	1.00	1.2970	-0.1144	0.4486	0.6791
18	1.10	1.3312	-0.1374	0.4041	0.6489
19	1.20	1.3550	-0.1622	0.3622	0.6193
20	1.30	1.3698	-0.1883	0.3232	0.5903
21	1.40	1.3771	-0.2157	0.2871	0.5619
22	1.50	1.3780	-0.2441	0.2538	0.5343
23	1.60	1.3739	-0.2732	0.2234	0.5074
24	1.70	1.3656	-0.3028	0.1958	0.4813
25	1.80	1.3542	-0.3326	0.1708	0.4560
26	1.90	1.3404	-0.3624	0.1484	0.4315
27	2.00	1.3249	-0.3920	0.1283	0.4078
28	2.25	1.2813	-0.4627	0.0879	0.3524
29	2.50	1.2371	-0.5280	0.0589	0.3024
30	2.75	1.1961	-0.5866	0.0386	0.2578
31	3.00	1.1598	-0.6381	0.0248	0.2162
32	3.25	1.1288	-0.6827	0.0156	0.1836
33	3.50	1.1029	-0.7208	0.0096	0.1534
34	3.75	1.0817	-0.7529	0.0059	0.1274
35	4.00	1.0644	-0.7798	0.0035	0.1052
36	4.25	1.0505	-0.8022	0.0020	0.0863
37	4.50	1.0393	-0.8207	0.0011	0.0703
38	5.00	1.0238	-0.8456	0.0004	0.0461
39	5.50	1.0142	-0.8620	0.0001	0.0296
40	6.00	1.0083	-0.8726	0.0000	0.0187
41	6.50	1.0048	-0.8794	0.0000	0.0115
42	7.00	1.0027	-0.8836	0.0000	0.0070
43	7.50	1.0015	-0.8861	0.0000	0.0041
44	8.00	1.0008	-0.8876	0.0000	0.0024
45	8.50	1.0005	-0.8885	0.0000	0.0014
46	9.00	1.0002	-0.8890	0.0000	0.0008
47	9.50	1.0001	-0.8893	0.0000	0.0004
48	10.00	1.0001	-0.8895	0.0000	0.0002
49	10.50	1.0000	-0.8896	0.0000	0.0001
50	11.00	1.0000	-0.8896	0.0000	0.0001
51	11.50	1.0000	-0.8896	0.0000	0.0000

MEAN NUSSELT NO= .8003 MEAN SHERWOOD NO= .4809

Table 4.5 b : Tau = 0.8 : Velocity, Temperature and Concentration distribution at X = 1.0 for Pr = 0.7, Sc = 0.2, N = 2 and U_∞ = 1.0 .

K	Y	U	V	THETA	CONC
1	0.03	0.0780	-0.0005	0.9836	0.9907
2	0.06	0.1533	-0.0014	0.9673	0.9813
3	0.09	0.2259	-0.0027	0.9509	0.9720
4	0.12	0.2960	-0.0045	0.9345	0.9626
5	0.15	0.3635	-0.0068	0.9181	0.9533
6	0.18	0.4285	-0.0094	0.9018	0.9439
7	0.21	0.4910	-0.0126	0.8855	0.9345
8	0.24	0.5510	-0.0162	0.8691	0.9253
9	0.27	0.6087	-0.0202	0.8528	0.9159
10	0.30	0.6640	-0.0246	0.8365	0.9066
11	0.40	0.8315	-0.0442	0.7825	0.8756
12	0.50	0.9748	-0.0684	0.7290	0.8446
13	0.60	1.0957	-0.0970	0.6763	0.8138
14	0.70	1.1959	-0.1296	0.6247	0.7832
15	0.80	1.2772	-0.1659	0.5745	0.7529
16	0.90	1.3416	-0.2056	0.5259	0.7228
17	1.00	1.3909	-0.2483	0.4792	0.6932
18	1.10	1.4268	-0.2935	0.4346	0.6640
19	1.20	1.4511	-0.3408	0.3923	0.6352
20	1.30	1.4655	-0.3897	0.3525	0.6070
21	1.40	1.4715	-0.4398	0.3153	0.5794
22	1.50	1.4706	-0.4906	0.2807	0.5524
23	1.60	1.4640	-0.5419	0.2467	0.5261
24	1.70	1.4529	-0.5930	0.2194	0.5005
25	1.80	1.4383	-0.6435	0.1927	0.4755
26	1.90	1.4211	-0.6939	0.1686	0.4515
27	2.00	1.4021	-0.7431	0.1468	0.4282
28	2.25	1.3494	-0.8574	0.1023	0.3733
29	2.50	1.2964	-0.9611	0.0697	0.3235
30	2.75	1.2472	-1.0533	0.0465	0.2787
31	3.00	1.2036	-1.1341	0.0304	0.2387
32	3.25	1.1661	-1.2040	0.0195	0.2033
33	3.50	1.1345	-1.2639	0.0123	0.1722
34	3.75	1.1082	-1.3148	0.0076	0.1451
35	4.00	1.0866	-1.3579	0.0046	0.1215
36	4.25	1.0690	-1.3940	0.0027	0.1014
37	4.50	1.0546	-1.4242	0.0015	0.0840
38	5.00	1.0342	-1.4660	0.0005	0.0571
39	5.50	1.0211	-1.4942	0.0002	0.0380
40	6.00	1.0128	-1.5130	0.0001	0.0249
41	6.50	1.0077	-1.5253	0.0000	0.0160
42	7.00	1.0045	-1.5331	0.0000	0.0100
43	7.50	1.0026	-1.5380	0.0000	0.0062
44	8.00	1.0015	-1.5411	0.0000	0.0038
45	8.50	1.0008	-1.5429	0.0000	0.0022
46	9.00	1.0005	-1.5439	0.0000	0.0013
47	9.50	1.0003	-1.5446	0.0000	0.0008
48	10.00	1.0001	-1.5449	0.0000	0.0004
49	10.50	1.0001	-1.5451	0.0000	0.0002
50	11.00	1.0000	-1.5452	0.0000	0.0001
51	11.50	1.0000	-1.5452	0.0000	0.0000

MEAN NUSSELT NO= .7649 MEAN SHERWOOD NO= .4698

Time required to reach steady state = 2.2

Table 4.5 c: Steady state velocity, Temperature & Concentration distributions at $X=1.0$ for $Pr=0.7, Sc=0.2, M=2.0$ and $U_{\infty}=1.0$

K	Y	U	V	THET	CON
1	0.01	0.2363	0.0000	0.9811	0.9897
2	0.02	0.4599	0.0000	0.9623	0.9794
3	0.03	0.6864	0.0001	0.9435	0.9690
4	0.04	0.9154	0.0001	0.9248	0.9587
5	0.05	1.1407	0.0002	0.9061	0.9485
6	0.06	1.3639	0.0002	0.8876	0.9382
7	0.07	1.5849	0.0003	0.8692	0.9279
8	0.08	1.8032	0.0004	0.8509	0.9177
9	0.09	2.0189	0.0005	0.8328	0.9075
0	0.10	2.2316	0.0006	0.8149	0.8973
1	0.14	3.0515	0.0013	0.7449	0.8570
2	0.18	3.8148	0.0022	0.6784	0.8173
3	0.22	4.5180	0.0032	0.6157	0.7784
4	0.26	5.1601	0.0046	0.5571	0.7405
5	0.30	5.7424	0.0061	0.5026	0.7035
6	0.34	6.2671	0.0080	0.4521	0.6675
7	0.38	6.7374	0.0100	0.4057	0.6327
8	0.42	7.1571	0.0124	0.3631	0.5990
9	0.46	7.5300	0.0149	0.3242	0.5665
0	0.50	7.8602	0.0177	0.2888	0.5352
1	0.60	8.5159	0.0259	0.2146	0.4622
2	0.70	8.9859	0.0346	0.1573	0.3966
3	0.80	9.3175	0.0433	0.1138	0.3382
4	0.90	9.5478	0.0514	0.0813	0.2866
5	1.00	9.7054	0.0583	0.0573	0.2414
6	1.10	9.8116	0.0639	0.0397	0.2019
7	1.20	9.8822	0.0681	0.0271	0.1677
8	1.30	9.9287	0.0709	0.0181	0.1382
9	1.40	9.9593	0.0727	0.0116	0.1129
0	1.50	9.9799	0.0735	0.0069	0.0913
1	2.00	9.9966	0.0725	0.0010	0.0320
2	2.50	10.0001	0.0718	0.0002	0.0101
3	3.00	10.0001	0.0716	0.0000	0.0029
4	3.50	10.0000	0.0715	0.0000	0.0008
5	4.00	10.0000	0.0715	0.0000	0.0002
6	4.50	10.0000	0.0715	0.0000	0.0000

NUSSELT NO=2.0665 MEAN SHERWOOD NO=1.2686

e 4.6 a : Tau = 0.1 : Velocity,Temperature and Concentration distribution at X = 1.0 for Pr = 0.7,Sc = 0.2,N =2 and U_∞ = 10.0 .

K	Y	U	V	THET	CON
1	0.01	0.1509	0.0001	0.9877	0.9928
2	0.02	0.3014	0.0002	0.9755	0.9856
3	0.03	0.4516	0.0003	0.9632	0.9784
4	0.04	0.6013	0.0006	0.9510	0.9711
5	0.05	0.7505	0.0009	0.9388	0.9639
6	0.06	0.8991	0.0012	0.9265	0.9567
7	0.07	1.0472	0.0016	0.9144	0.9495
8	0.08	1.1947	0.0021	0.9022	0.9423
9	0.09	1.3414	0.0026	0.8900	0.9351
10	0.10	1.4874	0.0032	0.8779	0.9279
11	0.14	2.0638	0.0065	0.8297	0.8992
12	0.18	2.6250	0.0108	0.7822	0.8705
13	0.22	3.1667	0.0162	0.7355	0.8420
14	0.26	3.6927	0.0226	0.6898	0.8137
15	0.30	4.1953	0.0302	0.6453	0.7855
16	0.34	4.6751	0.0388	0.6021	0.7576
17	0.38	5.1309	0.0487	0.5603	0.7299
18	0.42	5.5622	0.0596	0.5200	0.7026
19	0.46	5.9685	0.0716	0.4813	0.6755
20	0.50	6.3496	0.0847	0.4443	0.6489
21	0.60	7.1924	0.1232	0.3593	0.5839
22	0.70	7.8856	0.1655	0.2856	0.5219
23	0.80	8.4419	0.2090	0.2229	0.4633
24	0.90	8.6774	0.2509	0.1709	0.4063
25	1.00	9.2097	0.2869	0.1285	0.3572
26	1.10	9.4571	0.3213	0.0947	0.3102
27	1.20	9.6369	0.3473	0.0684	0.2673
28	1.30	9.7650	0.3668	0.0481	0.2265
29	1.40	9.8553	0.3804	0.0327	0.1938
30	1.50	9.9196	0.3869	0.0210	0.1629
31	2.00	9.9912	0.3921	0.0036	0.0646
32	2.50	9.9995	0.3911	0.0006	0.0225
33	3.00	10.0001	0.3906	0.0001	0.0070
34	3.50	10.0001	0.3904	0.0000	0.0020
35	4.00	10.0000	0.3903	0.0000	0.0005
36	4.50	10.0000	0.3903	0.0000	0.0001
37	5.00	10.0000	0.3903	0.0000	0.0000

MEAN NUSSELT NO=1.5531 MEAN SHERWOOD NO=1.0444

Table 4.6 b : Tau = 0.2 : Velocity,Temperature and Concentration distribution at X = 1.0 for Pr = 0.7,Sc = 0.2,N =2 and U_∞ = 10.0 .

Y	U	V	THET	CON
0.01	0.1045	0.0001	0.9915	0.9942
0.02	0.2086	0.0004	0.9830	0.9884
0.03	0.3125	0.0008	0.9744	0.9827
0.04	0.4160	0.0014	0.9659	0.9769
0.05	0.5192	0.0021	0.9574	0.9711
0.06	0.6221	0.0029	0.9489	0.9653
0.07	0.7247	0.0039	0.9404	0.9595
0.08	0.8269	0.0050	0.9319	0.9538
0.09	0.9288	0.0063	0.9234	0.9480
0.10	1.0303	0.0077	0.9149	0.9422
0.14	1.4329	0.0156	0.8810	0.9191
0.18	1.8294	0.0257	0.8471	0.8960
0.22	2.2194	0.0380	0.8135	0.8729
0.26	2.6025	0.0526	0.7800	0.8499
0.30	2.9783	0.0693	0.7468	0.8270
0.34	3.3462	0.0883	0.7139	0.8041
0.38	3.7058	0.1093	0.6813	0.7812
0.42	4.0567	0.1323	0.6492	0.7585
0.46	4.3983	0.1573	0.6175	0.7359
0.50	4.7303	0.1840	0.5864	0.7134
0.60	5.5161	0.2605	0.5111	0.6578
0.70	6.2327	0.3437	0.4402	0.6034
0.80	6.8757	0.4300	0.3747	0.5504
0.90	7.4424	0.5157	0.3149	0.4993
1.00	7.9331	0.5974	0.2614	0.4504
1.10	8.3502	0.6721	0.2142	0.4038
1.20	8.6990	0.7380	0.1733	0.3598
1.30	8.9864	0.7941	0.1384	0.3187
1.40	9.2207	0.8402	0.1091	0.2806
1.50	9.4107	0.8770	0.0846	0.2455
2.00	9.8218	0.9296	0.0265	0.1167
2.50	9.9451	0.9433	0.0082	0.0497
3.00	9.9827	0.9469	0.0026	0.0194
3.50	9.9945	0.9479	0.0008	0.0071
4.00	9.9982	0.9482	0.0003	0.0025
4.50	9.9994	0.9483	0.0001	0.0009
5.00	9.9998	0.9483	0.0000	0.0003
5.50	9.9999	0.9483	0.0000	0.0001
6.00	10.0000	0.9483	0.0000	0.0000

NUSSELT NO=1.0915 MEAN SHERWOOD NO= .8164

4.6 c : Tau = 0.6 : Velocity,Temperature and Concentration distribution at $X = 1.0$ for $Pr = 0.7, Sc = 0.2, N = 2$ and $U_\infty = 10.0$.

K	Y	U	V	THETA	CONC
1	0.01	0.0945	0.0002	0.9923	0.9945
2	0.02	0.1887	0.0006	0.9846	0.9891
3	0.03	0.2826	0.0013	0.9768	0.9836
4	0.04	0.3762	0.0021	0.9691	0.9781
5	0.05	0.4695	0.0032	0.9614	0.9727
6	0.06	0.5625	0.0045	0.9537	0.9672
7	0.07	0.6552	0.0060	0.9460	0.9617
8	0.08	0.7476	0.0077	0.9383	0.9563
9	0.09	0.8398	0.0096	0.9305	0.9508
10	0.10	0.9316	0.0118	0.9228	0.9453
11	0.14	1.2960	0.0237	0.8920	0.9235
12	0.18	1.6558	0.0391	0.8612	0.9016
13	0.22	2.0107	0.0579	0.8304	0.8798
14	0.26	2.3607	0.0800	0.7997	0.8580
15	0.30	2.7056	0.1054	0.7692	0.8362
16	0.34	3.0452	0.1340	0.7388	0.8144
17	0.38	3.3793	0.1657	0.7085	0.7927
18	0.42	3.7074	0.2004	0.6785	0.7711
19	0.46	4.0293	0.2380	0.6487	0.7495
20	0.50	4.3445	0.2783	0.6193	0.7280
21	0.60	5.1020	0.3937	0.5473	0.6748
22	0.70	5.8093	0.5199	0.4784	0.6224
23	0.80	6.4600	0.6521	0.4133	0.5712
24	0.90	7.0489	0.7851	0.3528	0.5214
25	1.00	7.5724	0.9142	0.2974	0.4733
26	1.10	8.0292	1.0350	0.2475	0.4272
27	1.20	8.4206	1.1442	0.2034	0.3833
28	1.30	8.7507	1.2397	0.1649	0.3420
29	1.40	9.0252	1.3209	0.1320	0.3033
30	1.50	9.2512	1.3863	0.1041	0.2673
31	2.00	9.7686	1.4971	0.0337	0.1313
32	2.50	9.9284	1.5288	0.0106	0.0576
33	3.00	9.9775	1.5360	0.0034	0.0231
34	3.50	9.9928	1.5407	0.0011	0.0087
35	4.00	9.9977	1.5415	0.0003	0.0031
36	4.50	9.9993	1.5418	0.0001	0.0011
37	5.00	9.9998	1.5418	0.0000	0.0004
38	5.50	9.9999	1.5419	0.0000	0.0001
39	6.00	10.0000	1.5419	0.0000	0.0000

MEAN NUSSELT NO=1.0456 MEAN SHERWOOD NO= .7971

Time required to reach steady state = 1.8

Table 4.6 d : Steady state Velocity ,Temperature & Concentration distributions at $X=1.0$ for $Pr=0.7, Sc=0.2, N=2.0$ and $U_\infty = 10.0$

K	Y	U	V	THETA	CONC
1	0.05	0.0438	-0.0007	0.9815	0.9698
2	0.10	0.0851	-0.0020	0.9631	0.9395
3	0.15	0.1241	-0.0039	0.9446	0.9093
4	0.20	0.1607	-0.0065	0.9261	0.8791
5	0.25	0.1950	-0.0098	0.9077	0.8490
6	0.30	0.2271	-0.0137	0.8892	0.8190
7	0.35	0.2570	-0.0182	0.8708	0.7890
8	0.40	0.2847	-0.0234	0.8524	0.7592
9	0.45	0.3104	-0.0292	0.8341	0.7296
10	0.50	0.3342	-0.0356	0.8158	0.7002
11	0.65	0.3936	-0.0602	0.7613	0.6136
12	0.80	0.4374	-0.0899	0.7076	0.5307
13	0.95	0.4671	-0.1242	0.6550	0.4528
14	1.10	0.4846	-0.1627	0.6039	0.3808
15	1.25	0.4915	-0.2049	0.5546	0.3157
16	1.40	0.4897	-0.2501	0.5073	0.2580
17	1.55	0.4806	-0.2977	0.4623	0.2079
18	1.70	0.4657	-0.3471	0.4198	0.1652
19	1.85	0.4464	-0.3976	0.3799	0.1296
20	2.00	0.4238	-0.4487	0.3426	0.1002
21	2.30	0.3722	-0.5496	0.2764	0.0586
22	2.60	0.3182	-0.6458	0.2207	0.0331
23	2.90	0.2603	-0.7346	0.1747	0.0182
24	3.20	0.2189	-0.8144	0.1374	0.0099
25	3.50	0.1773	-0.8847	0.1074	0.0053
26	3.80	0.1418	-0.9452	0.0835	0.0029
27	4.10	0.1121	-0.9965	0.0646	0.0015
28	4.40	0.0877	-1.0393	0.0498	0.0008
29	4.70	0.0680	-1.0745	0.0382	0.0004
30	5.00	0.0523	-1.1031	0.0292	0.0002
31	5.30	0.0399	-1.1261	0.0222	0.0001
32	5.60	0.0302	-1.1443	0.0168	0.0001
33	5.90	0.0227	-1.1587	0.0127	0.0000
34	6.20	0.0169	-1.1698	0.0095	0.0000
35	6.50	0.0125	-1.1784	0.0071	0.0000
36	6.80	0.0092	-1.1850	0.0053	0.0000
37	7.10	0.0067	-1.1900	0.0039	0.0000
38	7.40	0.0049	-1.1937	0.0029	0.0000
39	7.70	0.0035	-1.1965	0.0021	0.0000
40	8.00	0.0025	-1.1986	0.0015	0.0000
41	8.50	0.0014	-1.2006	0.0009	0.0000
42	9.00	0.0008	-1.2013	0.0005	0.0000
43	9.50	0.0004	-1.2025	0.0003	0.0000
44	10.00	0.0002	-1.2029	0.0002	0.0000
45	10.50	0.0001	-1.2031	0.0001	0.0000
46	11.00	0.0001	-1.2032	0.0000	0.0000
47	11.50	0.0000	-1.2032	0.0000	0.0000

MEAN NUSSELT NO= .5445 MEAN SHERWOOD NO= .8964

Time required to reach steady state = 3.2

Table 4.7 :Steady state velocity, Temperature & Concentration distributions at X=1.0 for Pr=0.7, Sc=2.0, N=0.0 and U_∞=0.0

K	Y	U	V	THETA	CONC
1	0.03	0.0353	-0.0001	0.9873	0.9807
2	0.06	0.0697	-0.0002	0.9745	0.9613
3	0.09	0.1032	-0.0005	0.9618	0.9420
4	0.12	0.1358	-0.0008	0.9490	0.9227
5	0.15	0.1676	-0.0012	0.9363	0.9034
6	0.18	0.1986	-0.0017	0.9236	0.8842
7	0.21	0.2287	-0.0023	0.9109	0.8650
8	0.24	0.2580	-0.0030	0.8982	0.8458
9	0.27	0.2865	-0.0037	0.8855	0.8267
10	0.30	0.3142	-0.0045	0.8728	0.8077
11	0.40	0.4010	-0.0081	0.8306	0.7448
12	0.50	0.4796	-0.0126	0.7887	0.6831
13	0.60	0.5505	-0.0178	0.7472	0.6229
14	0.70	0.6142	-0.0238	0.7061	0.5647
15	0.80	0.6711	-0.0304	0.6657	0.5088
16	0.90	0.7218	-0.0377	0.6260	0.4556
17	1.00	0.7666	-0.0455	0.5871	0.4052
18	1.10	0.8061	-0.0538	0.5492	0.3581
19	1.20	0.8407	-0.0624	0.5123	0.3143
20	1.30	0.8708	-0.0714	0.4767	0.2740
21	1.40	0.8968	-0.0807	0.4423	0.2373
22	1.50	0.9192	-0.0900	0.4092	0.2040
23	1.60	0.9382	-0.0995	0.3776	0.1742
24	1.70	0.9544	-0.1090	0.3475	0.1478
25	1.80	0.9679	-0.1184	0.3189	0.1245
26	1.90	0.9791	-0.1277	0.2918	0.1041
27	2.00	0.9883	-0.1369	0.2663	0.0865
28	2.25	1.0037	-0.1582	0.2093	0.0534
29	2.50	1.0116	-0.1775	0.1618	0.0316
30	2.75	1.0146	-0.1944	0.1230	0.0184
31	3.00	1.0147	-0.2088	0.0920	0.0104
32	3.25	1.0132	-0.2207	0.0676	0.0057
33	3.50	1.0111	-0.2305	0.0489	0.0031
34	3.75	1.0088	-0.2382	0.0347	0.0016
35	4.00	1.0068	-0.2441	0.0242	0.0004
36	4.50	1.0036	-0.2509	0.0115	0.0002
37	5.00	1.0017	-0.2544	0.0052	0.0001
38	5.50	1.0008	-0.2561	0.0023	0.0000
39	6.00	1.0003	-0.2568	0.0009	0.0000
40	6.50	1.0001	-0.2572	0.0004	0.0000
41	7.00	1.0001	-0.2573	0.0001	0.0000
42	7.50	1.0000	-0.2573	0.0001	0.0000
43	8.00	1.0000	-0.2574	0.0000	0.0000
44	8.50	1.0000	-0.2574	0.0000	0.0000

MEAN NUSSELT NO= .6298 MEAN SHERWOOD NO= .8829

Time required to reach steady state = 2.0

Table 4.8 :Steady state Velocity,Temperature, Concentration distributions at X=1.0 for Pr=0.7,Sc=2.0,N=0.0 and ω =1.0

K	Y	U	V	THETA	CONC
1	0.01	0.0840	0.0002	0.9926	0.9908
2	0.02	0.1678	0.0007	0.9851	0.9817
3	0.03	0.2516	0.0015	0.9777	0.9725
4	0.04	0.3353	0.0025	0.9702	0.9633
5	0.05	0.4169	0.0037	0.9628	0.9541
6	0.06	0.5023	0.0052	0.9553	0.9450
7	0.07	0.5857	0.0070	0.9479	0.9358
8	0.08	0.6690	0.0090	0.9404	0.9266
9	0.09	0.7521	0.0112	0.9330	0.9175
10	0.10	0.8352	0.0137	0.9255	0.9083
11	0.14	1.1664	0.0276	0.8958	0.8717
12	0.18	1.4959	0.0454	0.8660	0.8351
13	0.22	1.8234	0.0672	0.8363	0.7987
14	0.26	2.1488	0.0929	0.8067	0.7624
15	0.30	2.4717	0.1223	0.7772	0.7253
16	0.34	2.7920	0.1555	0.7477	0.6906
17	0.38	3.1092	0.1924	0.7185	0.6552
18	0.42	3.4230	0.2327	0.6894	0.6203
19	0.46	3.7329	0.2764	0.6605	0.5859
20	0.50	4.0384	0.3232	0.6319	0.5522
21	0.60	4.7811	0.4578	0.5617	0.4712
22	0.70	5.4855	0.6056	0.4941	0.3961
23	0.80	6.1437	0.7614	0.4299	0.3282
24	0.90	6.7484	0.9196	0.3696	0.2681
25	1.00	7.2940	1.0747	0.3140	0.2164
26	1.10	7.7771	1.2215	0.2635	0.1726
27	1.20	8.1971	1.3561	0.2184	0.1364
28	1.30	8.5560	1.4758	0.1787	0.1063
29	1.40	8.8583	1.5795	0.1444	0.0830
30	1.50	9.1101	1.6673	0.1150	0.0638
31	2.00	9.7132	1.8227	0.0384	0.0205
32	2.50	9.9081	1.8727	0.0124	0.0065
33	3.00	9.9703	1.8885	0.0040	0.0022
34	3.50	9.9903	1.8936	0.0013	0.0007
35	4.00	9.9968	1.8953	0.0004	0.0002
36	4.50	9.9989	1.8959	0.0001	0.0001
37	5.00	9.9997	1.8960	0.0000	0.0000
38	5.50	9.9999	1.8961	0.0000	0.0000
39	6.00	10.0000	1.8961	0.0000	0.0000

MEAN NUSSELT NO=1.0192 MEAN SHERWOOD NO=1.1980

Time required to reach steady state = 2.0

Table 4.9 : Steady state Velocity, Temperature & Concentration distributions at X=1.0 for Pr=0.7, Sc=2.0, N=0.0 and U_∞=10.0

K	Y	U	V	THET	CON
1	0.05	0.0119	0.0000	0.8295	0.7298
2	0.10	0.0187	0.0000	0.6879	0.5326
3	0.15	0.0221	0.0000	0.5705	0.3886
4	0.20	0.0232	0.0000	0.4730	0.2835
5	0.25	0.0229	0.0000	0.3921	0.2067
6	0.30	0.0217	0.0000	0.3249	0.1506
7	0.35	0.0200	0.0000	0.2691	0.1096
8	0.40	0.0161	0.0000	0.2227	0.0795
9	0.45	0.0161	0.0000	0.1841	0.0574
10	0.50	0.0142	0.0000	0.1519	0.0410
11	0.65	0.0093	0.0000	0.0873	0.0164
12	0.80	0.0058	0.0000	0.0501	0.0066
13	0.95	0.0036	0.0000	0.0288	0.0026
14	1.10	0.0022	0.0000	0.0166	0.0010
15	1.25	0.0013	0.0000	0.0095	0.0004
16	1.40	0.0008	0.0000	0.0055	0.0002
17	1.55	0.0004	0.0000	0.0031	0.0001
18	1.70	0.0003	0.0000	0.0018	0.0000
19	1.85	0.0002	0.0000	0.0010	0.0000
20	2.00	0.0001	0.0000	0.0006	0.0000
21	2.30	0.0000	0.0000	0.0002	0.0000
22	2.60	0.0000	0.0000	0.0001	0.0000
23	2.90	0.0000	0.0000	0.0000	0.0000

MEAN NUSSELT NO=3.6796 MEAN SHERWOOD NO=6.0976

Table 4.10 a : Tau = 0.05 : Velocity,Temperature and Concentration distribution at $X = 1.0$ for $Pr = 0.7, Sc = 2.0, N = 2.0$ and $U_\infty = 0.0$.

K	Y	U	V	THET	CON
1	0.05	0.0379	0.0000	0.9539	0.9220
2	0.10	0.0691	0.0000	0.9081	0.8449
3	0.15	0.0941	0.0000	0.8625	0.7695
4	0.20	0.1137	0.0000	0.8175	0.6965
5	0.25	0.1284	0.0000	0.7732	0.6266
6	0.30	0.1390	0.0000	0.7298	0.5604
7	0.35	0.1460	0.0000	0.6873	0.4963
8	0.40	0.1499	0.0000	0.6460	0.4405
9	0.45	0.1512	0.0000	0.6058	0.3873
10	0.50	0.1503	0.0000	0.5671	0.3367
11	0.65	0.1375	0.0000	0.4592	0.2203
12	0.80	0.1175	0.0000	0.3656	0.1379
13	0.95	0.0959	0.0000	0.2863	0.0835
14	1.10	0.0755	0.0000	0.2209	0.0491
15	1.25	0.0579	0.0000	0.1680	0.0281
16	1.40	0.0434	0.0000	0.1261	0.0157
17	1.55	0.0320	0.0000	0.0935	0.0086
18	1.70	0.0232	0.0000	0.0684	0.0046
19	1.85	0.0165	0.0000	0.0495	0.0024
20	2.00	0.0116	0.0000	0.0353	0.0012
21	2.30	0.0057	0.0000	0.0179	0.0003
22	2.60	0.0027	0.0000	0.0088	0.0001
23	2.90	0.0013	0.0000	0.0043	0.0000
24	3.20	0.0006	0.0000	0.0020	0.0000
25	3.50	0.0003	0.0000	0.0009	0.0000
26	3.80	0.0001	0.0000	0.0004	0.0000
27	4.10	0.0001	0.0000	0.0002	0.0000
28	4.40	0.0000	0.0000	0.0001	0.0000
29	4.70	0.0000	0.0000	0.0000	0.0000

MEAN NUSSELT NO=1.0414 MEAN SHERWOOD NO=1.76

Table 4.10 b : Tau = 0.3 : Velocity,Temperat and Concentration distribution a $X = 1.0$ for $Pr = 0.7, Sc = 2.0, N$ and $U_\infty = 0.0$.

K	Y	U	V	THET	CON
1	0.05	0.0869	-0.0009	0.9783	0.9638
2	0.10	0.1665	-0.0026	0.9556	0.9277
3	0.15	0.2392	-0.0051	0.9348	0.8916
4	0.20	0.3053	-0.0085	0.9132	0.8557
5	0.25	0.3649	-0.0128	0.8915	0.8199
6	0.30	0.4185	-0.0179	0.8699	0.7844
7	0.35	0.4663	-0.0238	0.8484	0.7492
8	0.40	0.5086	-0.0306	0.8269	0.7141
9	0.45	0.5457	-0.0382	0.8056	0.6800
10	0.50	0.5779	-0.0466	0.7843	0.6452
11	0.65	0.6465	-0.0768	0.7214	0.5483
12	0.80	0.6809	-0.1172	0.6601	0.4577
13	0.95	0.6882	-0.1609	0.6011	0.3761
14	1.10	0.6746	-0.2089	0.5446	0.3042
15	1.25	0.6459	-0.2598	0.4912	0.2425
16	1.40	0.6069	-0.3125	0.4410	0.1906
17	1.55	0.5615	-0.3657	0.3943	0.1479
18	1.70	0.5129	-0.4184	0.3511	0.1134
19	1.85	0.4634	-0.4696	0.3115	0.0859
20	2.00	0.4149	-0.5188	0.2753	0.0644
21	2.30	0.3244	-0.6052	0.2130	0.0359
22	2.60	0.2476	-0.6780	0.1628	0.0195
23	2.90	0.1853	-0.7374	0.1230	0.0104
24	3.20	0.1364	-0.7846	0.0920	0.0055
25	3.50	0.0991	-0.8212	0.0680	0.0028
26	3.80	0.0711	-0.8491	0.0498	0.0014
27	4.10	0.0505	-0.8700	0.0362	0.0007
28	4.40	0.0355	-0.8855	0.0260	0.0004
29	4.70	0.0247	-0.8969	0.0185	0.0002
30	5.00	0.0171	-0.9051	0.0131	0.0001
31	5.30	0.0117	-0.9109	0.0092	0.0000
32	5.60	0.0080	-0.9151	0.0064	0.0000
33	5.90	0.0054	-0.9180	0.0044	0.0000
34	6.20	0.0036	-0.9200	0.0030	0.0000
35	6.50	0.0024	-0.9215	0.0021	0.0000
36	6.80	0.0016	-0.9224	0.0014	0.0000
37	7.10	0.0011	-0.9231	0.0009	0.0000
38	7.40	0.0007	-0.9235	0.0006	0.0000
39	7.70	0.0004	-0.9238	0.0004	0.0000
40	8.00	0.0003	-0.9240	0.0003	0.0000
41	8.50	0.0001	-0.9242	0.0001	0.0000
42	9.00	0.0001	-0.9243	0.0001	0.0000
43	9.50	0.0000	-0.9243	0.0000	0.0000

MEAN NUSSELT NO= .6304 MEAN SHERWOOD NO=1.0536

Table 4.10 c : Tau = 1.60 : Velocity, Temperature and Concentration distribution at $X = 1.0$ for $Pr = 0.7$, $Sc = 2.0$, $N = 2$ and $U_\infty = 0.0$.

K	Y	U	V	THETA	CONC
1	0.05	0.0899	-0.0013	0.9780	0.9632
2	0.10	0.1725	-0.0039	0.9559	0.9251
3	0.15	0.2481	-0.0078	0.9339	0.8895
4	0.20	0.3169	-0.0131	0.9118	0.8529
5	0.25	0.3793	-0.0195	0.8899	0.8164
6	0.30	0.4355	-0.0273	0.8679	0.7800
7	0.35	0.4857	-0.0362	0.8460	0.7439
8	0.40	0.5303	-0.0464	0.8242	0.7081
9	0.45	0.5695	-0.0577	0.8024	0.6727
10	0.50	0.6036	-0.0701	0.7808	0.6378
11	0.65	0.6769	-0.1170	0.7166	0.5364
12	0.80	0.7145	-0.1718	0.6541	0.4426
13	0.95	0.7234	-0.2331	0.5939	0.3589
14	1.10	0.7104	-0.2992	0.5364	0.2839
15	1.25	0.6815	-0.3684	0.4821	0.2209
16	1.40	0.6418	-0.4390	0.4313	0.1688
17	1.55	0.5954	-0.5096	0.3842	0.1257
18	1.70	0.5456	-0.5789	0.3409	0.0935
19	1.85	0.4950	-0.6400	0.3014	0.0679
20	2.00	0.4453	-0.7101	0.2655	0.0485
21	2.30	0.3528	-0.8229	0.2046	0.0245
22	2.60	0.2739	-0.9189	0.1562	0.0121
23	2.90	0.2095	-0.9985	0.1184	0.0059
24	3.20	0.1583	-1.0631	0.0892	0.0028
25	3.50	0.1184	-1.1148	0.0669	0.0011
26	3.80	0.0879	-1.1554	0.0499	0.0007
27	4.10	0.0647	-1.1871	0.0370	0.0003
28	4.40	0.0473	-1.2114	0.0273	0.0002
29	4.70	0.0343	-1.2300	0.0201	0.0001
30	5.00	0.0247	-1.2440	0.0147	0.0000
31	5.30	0.0177	-1.2545	0.0107	0.0000
32	5.60	0.0126	-1.2622	0.0078	0.0000
33	5.90	0.0089	-1.2680	0.0056	0.0000
34	6.20	0.0063	-1.2721	0.0040	0.0000
35	6.50	0.0044	-1.2752	0.0029	0.0000
36	6.80	0.0030	-1.2773	0.0020	0.0000
37	7.10	0.0021	-1.2789	0.0014	0.0000
38	7.40	0.0014	-1.2800	0.0010	0.0000
39	7.70	0.0010	-1.2808	0.0007	0.0000
40	8.00	0.0007	-1.2813	0.0005	0.0000
41	8.50	0.0003	-1.2818	0.0003	0.0000
42	9.00	0.0002	-1.2820	0.0001	0.0000
43	9.50	0.0001	-1.2822	0.0001	0.0000
44	10.00	0.0000	-1.2822	0.0000	0.0000

MEAN NUSSELT NO= .6337 MEAN SHERWOOD NO=1.0616

Time required to reach steady state = 2.80

Table 4.10 d: Steady state velocity, Temperature and Concentration distributions at $X=1.0$ for $Pr=0.7$, $Sc=2.0$, $N=2.0$ and $U_\infty=0.0$

K	Y	U	V	THETA	CONC
1	0.03	0.0652	-0.0003	0.9854	0.9778
2	0.06	0.1278	-0.0009	0.9709	0.9556
3	0.09	0.1878	-0.0019	0.9563	0.9335
4	0.12	0.2452	-0.0031	0.9418	0.9113
5	0.15	0.3002	-0.0047	0.9272	0.8892
6	0.18	0.3527	-0.0065	0.9127	0.8671
7	0.21	0.4029	-0.0087	0.8981	0.8450
8	0.24	0.4507	-0.0112	0.8836	0.8230
9	0.27	0.4964	-0.0139	0.8691	0.8010
10	0.30	0.5398	-0.0170	0.8546	0.7791
11	0.40	0.6691	-0.0303	0.8065	0.7068
12	0.50	0.7770	-0.0466	0.7588	0.6362
13	0.60	0.8655	-0.0655	0.7117	0.5679
14	0.70	0.9370	-0.0867	0.6655	0.5025
15	0.80	0.9934	-0.1099	0.6202	0.4407
16	0.90	1.0368	-0.1347	0.5762	0.3831
17	1.00	1.0691	-0.1606	0.5335	0.3299
18	1.10	1.0922	-0.1874	0.4924	0.2816
19	1.20	1.1076	-0.2146	0.4530	0.2382
20	1.30	1.1167	-0.2418	0.4154	0.1997
21	1.40	1.1209	-0.2688	0.3797	0.1660
22	1.50	1.1212	-0.2952	0.3460	0.1367
23	1.60	1.1187	-0.3209	0.3143	0.1117
24	1.70	1.1140	-0.3455	0.2846	0.0905
25	1.80	1.1079	-0.3689	0.2570	0.0727
26	1.90	1.1008	-0.3911	0.2313	0.0579
27	2.00	1.0932	-0.4119	0.2076	0.0457
28	2.25	1.0735	-0.4549	0.1567	0.0250
29	2.50	1.0557	-0.4893	0.1162	0.0133
30	2.75	1.0409	-0.5159	0.0847	0.0068
31	3.00	1.0293	-0.5360	0.0608	0.0034
32	3.25	1.0205	-0.5509	0.0429	0.0017
33	3.50	1.0140	-0.5617	0.0297	0.0008
34	3.75	1.0094	-0.5693	0.0202	0.0004
35	4.00	1.0061	-0.5746	0.0134	0.0002
36	4.50	1.0026	-0.5796	0.0059	0.0000
37	5.00	1.0010	-0.5818	0.0025	0.0000
38	5.50	1.0004	-0.5827	0.0010	0.0000
39	6.00	1.0001	-0.5831	0.0004	0.0000
40	6.50	1.0001	-0.5832	0.0001	0.0000
41	7.00	1.0000	-0.5832	0.0000	0.0000
42	7.50	1.0000	-0.5833	0.0000	0.0000

MEAN NUSSELT NO= .7103 MEAN SHERWOOD NO=1.0169

Time required to reach steady state = 2.40

Table 4.11 :Steady state velocity,Temperature and Concentration distributions at $X=1.0$ for $Pr=0.7, Sc=2.0, N=2.0$ and $U_\infty=1.0$

K	Y	U	V	THETA	CONC
1	0.01	0.4412	0.0000	0.9632	0.9387
2	0.02	0.8629	0.0000	0.9277	0.8811
3	0.03	1.2661	0.0000	0.8935	0.8270
4	0.04	1.6516	0.0000	0.8606	0.7762
5	0.05	2.0202	0.0001	0.8289	0.7286
6	0.06	2.3725	0.0001	0.7983	0.6838
7	0.07	2.7094	0.0001	0.7689	0.6418
8	0.08	3.0315	0.0001	0.7405	0.6024
9	0.09	3.3395	0.0002	0.7132	0.5653
10	0.10	3.6339	0.0002	0.6868	0.5306
11	0.14	4.6827	0.0004	0.5911	0.4127
12	0.18	5.5591	0.0007	0.5087	0.3212
13	0.22	6.2914	0.0011	0.4376	0.2501
14	0.26	6.9035	0.0015	0.3764	0.1949
15	0.30	7.4152	0.0020	0.3236	0.1521
16	0.34	7.8431	0.0026	0.2782	0.1188
17	0.38	8.2010	0.0032	0.2389	0.0930
18	0.42	8.5005	0.0039	0.2051	0.0728
19	0.46	8.7512	0.0047	0.1760	0.0571
20	0.50	8.9613	0.0055	0.1508	0.0448
21	0.60	9.3383	0.0078	0.1030	0.0254
22	0.70	9.5796	0.0101	0.0700	0.0147
23	0.80	9.7338	0.0123	0.0474	0.0087
24	0.90	9.8322	0.0143	0.0319	0.0052
25	1.00	9.8947	0.0159	0.0213	0.0031
26	1.10	9.9344	0.0172	0.0141	0.0019
27	1.20	9.9595	0.0182	0.0092	0.0011
28	1.30	9.9754	0.0190	0.0059	0.0007
29	1.40	9.9856	0.0194	0.0036	0.0004
30	1.50	9.9924	0.0197	0.0021	0.0002
31	2.00	9.9990	0.0199	0.0003	0.0000
32	2.50	9.9999	0.0199	0.0000	0.0000
33	3.00	10.0000	0.0199	0.0000	0.0000
34	3.50	10.0000	0.0199	0.0000	0.0000
35	4.00	10.0000	0.0199	0.0000	0.0000
36	4.50	10.0000	0.0199	0.0000	0.0000
37	5.00	10.0000	0.0199	0.0000	0.0000
38	5.50	10.0000	0.0199	0.0000	0.0000
39	6.00	10.0000	0.0199	0.0000	0.0000

MEAN NUSSELT NO=3.7932 MEAN SHERWOOD NO=6.1804

Table 4.12 a : Tau = 0.05 : Velocity,Temperature and Concentration distribution at $X = 1.0$ for $Pr = 0.7, Sc = 2.0, N = 2$ and $U_{\infty} = 10.0$.

K	Y	U	V	THET	CON
1	0.01	0.1488	0.0001	0.9878	0.9807
2	0.02	0.2973	0.0002	0.9755	0.9614
3	0.03	0.4453	0.0004	0.9633	0.9422
4	0.04	0.5929	0.0007	0.9511	0.9229
5	0.05	0.7401	0.0010	0.9389	0.9038
6	0.06	0.8867	0.0014	0.9267	0.8847
7	0.07	1.0327	0.0019	0.9146	0.8658
8	0.08	1.1781	0.0024	0.9024	0.8469
9	0.09	1.3229	0.0030	0.8903	0.8282
10	0.10	1.4669	0.0037	0.8782	0.8096
11	0.14	2.0354	0.0075	0.8301	0.7366
12	0.18	2.5893	0.0125	0.7827	0.6669
13	0.22	3.1262	0.0186	0.7361	0.6008
14	0.26	3.6439	0.0260	0.6906	0.5368
15	0.30	4.1410	0.0346	0.6462	0.4811
16	0.34	4.6159	0.0445	0.6031	0.4279
17	0.38	5.0677	0.0556	0.5614	0.3792
18	0.42	5.4956	0.0680	0.5212	0.3348
19	0.46	5.8992	0.0816	0.4826	0.2946
20	0.50	6.2784	0.0964	0.4456	0.2583
21	0.60	7.1195	0.1396	0.3608	0.1837
22	0.70	7.8146	0.1871	0.2872	0.1280
23	0.80	8.3755	0.2361	0.2245	0.0873
24	0.90	8.8175	0.2837	0.1724	0.0582
25	1.00	9.1573	0.3272	0.1299	0.0380
26	1.10	9.4124	0.3650	0.0960	0.0243
27	1.20	9.5996	0.3959	0.0694	0.0152
28	1.30	9.7345	0.4199	0.0489	0.0094
29	1.40	9.8309	0.4375	0.0333	0.0056
30	1.50	9.9003	0.4495	0.0215	0.0031
31	2.00	9.9852	0.4596	0.0037	0.0004
32	2.50	9.9978	0.4611	0.0006	0.0001
33	3.00	9.9997	0.4613	0.0001	0.0000
34	3.50	9.9999	0.4613	0.0000	0.0000
35	4.00	10.0000	0.4613	0.0000	0.0000
36	4.50	10.0000	0.4613	0.0000	0.0000
37	5.00	10.0000	0.4613	0.0000	0.0000
38	5.50	10.0000	0.4613	0.0000	0.0000
39	6.00	10.0000	0.4613	0.0000	0.0000

MEAN NUSSELT NO=1.5500 MEAN SHERWOOD NO=2.1264

Table 4.12 b : Tau = 0.2 : Velocity,Temperature and Concentration distribution at $X = 1.0$ for $Pr = 0.7, Sc = 2.0, N = 2$ and $U_{\infty} = 10.0$.

K	Y	U	V	THETA	CONC
1	0.01	0.0927	0.0002	0.9923	0.9905
2	0.02	0.1852	0.0007	0.9847	0.9810
3	0.03	0.2773	0.0014	0.9770	0.9715
4	0.04	0.3691	0.0023	0.9694	0.9620
5	0.05	0.4607	0.0034	0.9617	0.9525
6	0.06	0.5520	0.0047	0.9541	0.9430
7	0.07	0.6429	0.0063	0.9464	0.9335
8	0.08	0.7336	0.0081	0.9387	0.9240
9	0.09	0.8240	0.0102	0.9311	0.9145
10	0.10	0.9141	0.0124	0.9234	0.9051
11	0.14	1.2717	0.0250	0.8928	0.8671
12	0.18	1.6248	0.0413	0.8623	0.8293
13	0.22	1.9733	0.0610	0.8317	0.7916
14	0.26	2.3171	0.0844	0.8013	0.7541
15	0.30	2.6561	0.1112	0.7710	0.7169
16	0.34	2.9902	0.1414	0.7408	0.6800
17	0.38	3.3190	0.1749	0.7108	0.6436
18	0.42	3.6423	0.2116	0.6810	0.6078
19	0.46	3.9597	0.2514	0.6514	0.5726
20	0.50	4.2710	0.2940	0.6222	0.5381
21	0.60	5.0206	0.4165	0.5506	0.4557
22	0.70	5.7230	0.5508	0.4820	0.3806
23	0.80	6.3720	0.6921	0.4171	0.3122
24	0.90	6.9620	0.8351	0.3567	0.2530
25	1.00	7.4890	0.9746	0.3012	0.2025
26	1.10	7.9513	1.1061	0.2512	0.1603
27	1.20	8.3496	1.2259	0.2069	0.1257
28	1.30	8.6871	1.3318	0.1681	0.0978
29	1.40	8.9693	1.4228	0.1348	0.0754
30	1.50	9.2028	1.4992	0.1066	0.0575
31	2.00	9.7485	1.6309	0.0348	0.0161
32	2.50	9.9208	1.6723	0.0110	0.0058
33	3.00	9.9748	1.6852	0.0035	0.0019
34	3.50	9.9919	1.6893	0.0011	0.0006
35	4.00	9.9974	1.6906	0.0004	0.0002
36	4.50	9.9991	1.6910	0.0001	0.0001
37	5.00	9.9997	1.6911	0.0000	0.0000
38	5.50	9.9999	1.6912	0.0000	0.0000
39	6.00	10.0000	1.6912	0.0000	0.0000

MEAN NUSSELT NO=1.0417 MEAN SHERWOOD NO=1.2301

Time required to reach steady state = 1.80

Table 4.12 c : Steady state Velocity, Temperature and Concentration distributions at $x=1.0$ for $Pr=0.7, Sc=2.0, M=2.0$ and $U_{\infty}=10.0$

	TAU	MEAN NUSSELT NO.	MEAN SHERWOOD NO.
U_∞=0.0	.05	3.6780	1.9831
	.10	1.9317	1.0256
	.15	1.4747	0.7850
	.20	1.2554	0.6610
	.25	1.1219	0.5963
	.30	1.0315	0.5505
	.35	0.9659	0.5156
	.40	0.9163	0.4892
	.60	0.7318	0.3902
	.80	0.6498	0.3459
	1.00	0.6014	0.3192
	1.20	0.5720	0.3025
	1.40	0.5543	0.2920
	1.60	0.5444	0.2856
	1.80	0.5399	0.2819
	2.00	0.5387	0.2710
	2.20	0.5393	0.2789
	2.40	0.5406	0.2783
	2.60	0.5419	0.2778
	2.80	0.5431	0.2773
	3.00	0.5439	0.2769
	3.20	0.5445	0.2764
U_∞=1.0	.05	3.7166	2.0050
	.10	1.9369	1.0498
	.15	1.4932	0.8199
	.20	1.2824	0.7110
	.25	1.1569	0.6465
	.30	1.0738	0.6071
	.35	1.0150	0.5782
	.40	0.9716	0.5574
	.60	0.7885	0.4656
	.80	0.7155	0.4330
	1.00	0.6754	0.4171
	1.20	0.6547	0.4099
	1.40	0.6425	0.4050
	1.60	0.6357	0.4029
	1.80	0.6319	0.4018
	2.00	0.6298	0.4013
U_∞=10.	.05	3.7916	2.1382
	.10	2.0623	1.2692
	.15	1.6939	1.1006
	.20	1.5444	1.0394
	.40	1.1761	0.8501
	.60	1.0701	0.8037
	.80	1.0353	0.7869
	1.00	1.0240	0.7841
	1.20	1.0205	0.7826
	1.40	1.0195	0.7822
	1.60	1.0193	0.7820
	1.80	1.0192	0.7820
	2.00	1.0192	0.7820

for N = 0.0

	TAU	MEAN NUSSELT NO.	MEAN SHERWOOD NO.
U_∞=0.0	.05	3.0818	1.9654
	.10	1.9363	1.1295
	.15	1.4850	0.7914
	.20	1.2700	0.6777
	.25	1.1427	0.6095
	.30	1.0570	0.5650
	.35	0.9963	0.5334
	.40	0.9550	0.5103
	.60	0.8047	0.4370
	.80	0.7573	0.4039
	1.00	0.7432	0.3951
	1.20	0.7431	0.3934
	1.40	0.7472	0.3941
	1.60	0.7515	0.3952
	1.80	0.7547	0.396[illegible]
	2.00	0.7567	0.3966
	2.20	0.7570	0.3970
	2.40	0.7565	0.3972
	2.60	0.7563	0.3973
	2.80	0.7560	0.3973
U_∞=1.0	.05	3.7193	2.0667
	.10	1.9424	1.0531
	.15	1.5024	0.8255
	.20	1.2956	0.7129
	.25	1.1748	0.6594
	.30	1.0907	0.6227
	.35	1.0433	0.5950
	.40	1.0057	0.5775
	.60	0.8560	0.5015
	.80	0.8003	0.4[illegible]9
	1.00	0.7797	0.4730
	1.20	0.7716	0.4710
	1.40	0.7673	0.4701
	1.60	0.7658	0.4610
	1.80	0.7653	0.4606
	2.00	0.765[illegible]	0.46[illegible]
	2.20	0.7640	0.4628
U_∞=10.	.05	3.7932	2.1335
	.10	2.0605	1.2085
	.15	1.7001	1.1045
	.20	1.5531	1.1425
	.40	1.1327	0.9032
	.60	1.0815	0.8164
	.80	1.0595	0.8029
	1.00	1.0495	0.7980
	1.20	1.0406	0.7976
	1.40	1.0408	0.7974
	1.60	1.0456	0.7971
	1.80	1.0456	0.7971

for N = 2.0

Table 4.13 : Transient mean Nusselt and Sherwood numbers for Pr = 0.7, Sc = 0.2

For N = 0.0

	TAU	MEAN NUSSELT NO.	MEAN SHERWOOD NO.
U_∞=0.0	.05	3.6780	6.0951
	.10	1.9317	3.2991
	.15	1.4747	2.5045
	.20	1.2554	2.1283
	.25	1.1219	1.9009
	.30	1.0315	1.7471
	.35	0.9659	1.6351
	.40	0.9163	1.5501
	.60	0.7318	1.2355
	.80	0.6498	1.0933
	1.00	0.6014	1.0085
	1.20	0.5720	0.9558
	1.40	0.5543	0.9231
	1.60	0.5444	0.9038
	1.80	0.5399	0.8937
	2.00	0.5387	0.8896
	2.20	0.5393	0.8891
	2.40	0.5406	0.8904
	2.60	0.5419	0.8922
	2.80	0.5431	0.8940
	3.00	0.5439	0.8954
	3.20	0.5445	0.8964
U_∞=1.0	.05	3.7166	6.2014
	.10	1.9369	3.2506
	.15	1.4932	1.4790
	.20	1.2824	2.1100
	.25	1.1669	1.8873
	.30	1.0738	1.7377
	.35	1.0150	1.6303
	.40	0.9716	1.5493
	.60	0.7885	1.2286
	.80	0.7155	1.0863
	1.00	0.6764	1.0032
	1.20	0.6547	0.9527
	1.40	0.6425	0.9213
	1.60	0.6357	0.9018
	1.80	0.6319	0.8900
	2.00	0.6298	0.8829
U_∞=10.	.05	3.7918	6.1783
	.10	2.0623	3.1373
	.15	1.6939	2.4279
	.20	1.5414	2.1191
	.40	1.1761	1.5042
	.60	1.0791	1.3034
	.80	1.0353	1.2326
	1.00	1.0240	1.2097
	1.20	1.0206	1.2011
	1.40	1.0195	1.1983
	1.60	1.0193	1.1982
	1.80	1.0192	1.1981
	2.00	1.0192	1.1980

For N = 2.0

	TAU	MEAN NUSSELT NO.	MEAN SHERWOOD NO.
U_∞=0.0	.05	3.6796	6.0976
	.10	1.9344	3.3033
	.15	1.4791	2.5117
	.20	1.2613	2.1380
	.25	1.1297	1.9139
	.30	1.0414	1.6733
	.35	0.9779	1.6550
	.40	0.9305	1.5738
	.60	0.7585	1.2804
	.80	0.6891	1.1593
	1.00	0.6542	1.0975
	1.20	0.6379	1.0679
	1.40	0.6317	1.0562
	1.60	0.6304	1.0536
	1.80	0.6309	1.0548
	2.00	0.6318	1.0570
	2.20	0.6327	1.0590
	2.40	0.6332	1.0604
	2.60	0.6335	1.0613
	2.80	0.6337	1.0618
U_∞=1.0	.05	3.7191	6.2035
	.10	1.9397	3.2544
	.15	1.4976	2.4853
	.20	1.2685	2.1159
	.25	1.1650	1.8991
	.30	1.0839	1.7525
	.35	1.0273	1.6465
	.40	0.9863	1.5716
	.60	0.8172	1.2695
	.80	0.7570	1.1401
	1.00	0.7297	1.0825
	1.20	0.7175	1.0452
	1.40	0.7125	1.0320
	1.60	0.7107	1.0235
	1.80	0.7102	1.0196
	2.00	0.7102	1.0178
	2.20	0.7102	1.0171
	2.40	0.7103	1.0169
U_∞=10.	.05	3.7932	6.1804
	.10	2.0650	3.1405
	.15	1.6990	2.4335
	.20	1.5500	2.1204
	.40	1.1690	1.5207
	.60	1.0677	1.3269
	.80	1.0556	1.2606
	1.00	1.0457	1.2395
	1.20	1.0428	1.2325
	1.40	1.0420	1.2306
	1.60	1.0417	1.2302
	1.80	1.0417	1.2301

Table 4.14 : Transient mean Nusselt and Sherwood numbers for Pr = 0.7, Sc = 2.0

Chapter 5

CONCLUSIONS

A boundary-layer analysis for transient, laminar, combined forced and natural convection along an isothermal vertical flat plate subjected to a step change in temperature and concentration has been solved by a highly implicit finite-difference method. In order to obtain a solution to this problem, the coupled governing conservation equations must be solved simultaneously. The computer code in the Appendix is based on a non-uniform mesh in the direction normal to the plate (Y-direction). The parameters of the problem are : (i) the buoyancy ratio parameter, N, (ii) the Prandtl number, Pr, (iii) the Schmidt number, Sc, and (iv) the Forced-free convection parameter, $U_\infty = Re/Gr^{1/2}$. Results found for various values of the above parameters show the following :

a) During the initial transient period the heat transfer is by conduction only, and the mass transfer is by diffusion only, even for strong forced flow.

b) After the initial conduction-diffusion regime, combined buoyancy forces along with free stream velocity (for combined forced and free flow) generate the motion.

c) The transient velocity, temperature and concentration profiles for free convection show a temporal maximum

over their respective steady state values. However, this phenomenon of temporal maximum is not observed for combined free and forced convection.

d) The time required to reach the steady state decreases with increase in forced-free convection parameter U_∞.

e) Both Nusselt and Sherwood numbers pass through a temporal minimum before reaching their steady state values. However, it is observed only for free convective flow and not for combined free and forced convection.

f) For mass diffusion aiding the flow both mean Nusselt and Sherwood numbers are higher than those for N = 0.0 (Pure thermal convection).

g) Mean Nusselt and Sherwood numbers are higher for higher values of U_∞.

APPENDIX

COMPUTER PROGRAM LISTING

```
************************************************************************
C for transient,laminar,free-forced convection with heat and
C mass transfer from a isothermal plate.
************************************************************************

C Variables:-
C ----------
C    A(I)      lower diagonal elements in a TRIDIAGONAL matrix
C    B(I)      main diagnal elements in TRIDIAGONAL matrix
C    C(I)      upper diagonal elements in TRIDIAGONAL matrix
C    AU,AV,ATHET,ACON           : guess values in the iterative method
C    PU,PV,PTHET,PCON           : previous x-location values
C    UI,THETI,CONI              : values at the previous time
C    NUX,SHX            instantaneous local Nusselt and Sherwood
C                       numbers respectively
C    NUM,SHM            instantaneous mean Nusselt and Sherwood
C                       numbers respectively
C    PR        Pradtl number
C    SC        Schmidt number
C    RN        buoyancy ratio parameter
C    UINF      free-forced convection parameter
C    ALP       relaxation factor
C    DELY1,DELY2,DELY3,DELY4 : step sizes n Y direction
C    ALPHI                      : ratio of smaller step size to
C                                 larger step size in Y direction
C    DELX                       : step size in X direction
C    DETAU1,DETAU2              : time-steps
C    N         number of steps in X direction
C    M         number of steps in Y direction
C    IP,IQ,IR           points at change of mesh size

************************************************************************
C
      REAL NUX,NUM
      DIMENSION A(39),B(39),C(39),D(39),U(39),THET(39),AU(39)
      DIMENSION ATHET(39),PU(39,51),PTHET(39,51),V(39),AV(39)
      DIMENSION CON(39),ACON(39),PCON(39,51),Y(39)
      DIMENSION UI(39,51),THETI(39,51),CONI(39,51)
      DIMENSION NUX(51),SHX(51)
      OPEN (UNIT=21,DEVICE='DSK',FILE='FFFF.CDR')
      OPEN (UNIT=25,DEVICE='DSK',FILE='FFCC16.DAT')
      OPEN (UNIT=23,DEVICE='DSK',FILE='NS16.DAT')
C
C  Read in and print out the parameters
```

```
C
      READ(21,*)N,IP,IQ,IR,PR,THEO,SMALL,SC,RN,CONO,SSMALL
      READ(21,*)DELY1,DELY2,DELY3,DELY4,DTAU1,DTAU2
      READ(21,*)DELX,M,ALP,UINF
      WRITE(25,111)N,M,DELX,DELY1,DELY2,DELY3,DELY4,DTAU1,DTAU2
     1,SMALL
111   FORMAT(2X,'NO OF STEPS IN Y DIRECTION = ',I3,/
     1,2X,'NO OF STEPS IN X DIRECTION = ',I3,/
     2,2X,'STEP SIZE IN X DIRECTION = ',F4.2,/
     3,2X,'STEP SIZES IN Y DIRECTION = ',F4.2,1X,F4.2,1X,F4.2,'&'
     4,F4.2,/,2X,'TIME STEP = ',F4.2,2X,F4.2,'&',F4.2,2X
     5,'CONVERGENCE CRITERIA IS EPS LESS THAN ',F7.5)
      WRITE(25,94) PR,SC,RN,UINF
94    FORMAT(2X,'For',3X,'Prandtl NO. =',F4.2,2X,',',2X
     1,'Schmidt NO. =',F4.2,/,2X,'Parameter N =',F4.2
     2,2X,'&',2X,'Re/(Gr)**0.5  = ',F5.2)
      WRITE(25,95)
95    FORMAT(1X,50(1H-),/)
      WRITE(23,888)
888   FORMAT(4X,'TAU',5X,'MEAN NUSSELT NO.' ,4X,'MEAN SHERWOOD NO.')
      WRITE(23,887)
887   FORMAT(2X,50(1H-))
*
*  Values of all the constants required to be calculated repeatedly
*  calculated here
*
        DY2=0.5/DELY2$DY1=0.5/DELY1$DTAU=DTAU1$DT=1.0/DTAU
        DY3=0.5/DELY3$DY4=0.5/DELY4
        DELX=1./DELX$D1=DELY1*DELX$D2=DELY2*DELX$D3=DELY3*DELX
        D4=DELY4*DELX
        DYM2=4.0*DY2*DY2$DYM1=4.0*DY1*DY1$DYE2=DYM2/PR$DYE1=DYM1/PR
        DYM3=4.0*DY3*DY3$DYM4=4.0*DY4*DY4$DYE3=DYM3/PR$DYE4=DYM4/PR
        DYC2=DYM2/SC$DYC1=DYM1/SC
        DYC3=DYM3/SC$DYC4=DYM4/SC
        APHI=DELY1/DELY2$BPHI=DELY2/DELY3$CPHI=DELY3/DELY4
        APHI1=2.*APHI*APHI/(1.0+APHI)$APHI2=(APHI-1.)/(APHI+1.)
        APHI3=2.*(1.-APHI)
        BPHI1=2.*BPHI*BPHI/(1.0+BPHI)$BPHI2=(BPHI-1.)/(BPHI+1.)
        BPHI3=2.*(1.-BPHI)
        CPHI1=2.*CPHI*CPHI/(1.0+CPHI)$CPHI2=(CPHI-1.)/(CPHI+1.)
        CPHI3=2.*(1.-CPHI)
        TAU=0.0
        IP2=IP+1$IQ2=IQ+1$IR2=IR+1
        Y(1)=DELY1
```

```
      DO 180 K=2,IP
      Y(K)=Y(K-1)+DELY1
180   CONTINUE
      DO 190 K=IP2,IQ
      Y(K)=Y(K-1)+DELY2
190   CONTINUE
      DO 191 K=IQ2,IR
      Y(K)=Y(K-1)+DELY3
191   CONTINUE
      DO 192 K=IR2,N
      Y(K)=Y(K-1)+DELY4
192   CONTINUE
*
*  initial and boundary conditions are generated and initial guesses
*  for U's,V's,Theta's and C's at 1,an iteration counter ,in the
*  iterative procedure (cf. section 3.3) are given
*
     DO 10 K=1,N
     AU(K)=UINF$AV(K)=0.0$ATHET(K)=EXP(-1.9*Y(K))
     ACON(K)=EXP(-1.9*Y(K))
     PU(K,1)=UINF$PTHET(K,1)=0.0$PCON(K,1)=0.0
     DO 10 L=1,M
     UI(K,L)=UINF$THETI(K,L)=0.0$CONI(K,L)=0.0
10   CONTINUE
C
C  marching in time
C
125 TAU=TAU+DTAU
    WRITE(5,*)TAU
C
C  for higher times relaxation factor - ALP is increased
C
    IF(TAU.GE.1.)ALP=1.0
C  marching in X - direction (downstream)
    J=0 $X=0.0
    DO 140 L=1,M
    X=X+1.0/DELX
    J=J+1
    ITR=0
400 ITR=ITR+1
*
* elements of TRIDIAGONAL coefficient matrix and right hand side
* vector for momentum equation (cf. eqn. 3.17)are calculated
*
```

```
      DO 20 K=1,IP
      PROD=AV(K)*DY1
      A(K)=-DYM1-PROD
      B(K)=AU(K)*DELX+2.0*DYM1+DT
      C(K)=-DYM1+PROD
 20   CONTINUE
      A(IP)=A(IP)+APHI2*C(IP)
      B(IP)=B(IP)+APHI3*C(IP)
      C(IP)=C(IP)*APHI1
*
*   step size is changed from smaller step size DELY1 to larger step
*   size DELY2
*
      DO 30 K=IP2,IQ
      PROD=AV(K)*DY2
      A(K)=-DYM2-PROD
      B(K)=AU(K)*DELX+2.0*DYM2+DT
      C(K)=-DYM2+PROD
 30   CONTINUE
      A(IQ)=A(IQ)+BPHI2*C(IQ)
      B(IQ)=B(IQ)+BPHI3*C(IQ)
      C(IQ)=C(IQ)*BPHI1
*
*   step size is changed from smaller step size DELY2 to larger step
*   size DELY3
*
      DO 21 K=IQ2,IR
      PROD=AV(K)*DY3
      A(K)=-DYM3-PROD
      B(K)=AU(K)*DELX+2.0*DYM3+DT
      C(K)=-DYM3+PROD
 21   CONTINUE
      A(IR)=A(IR)+CPHI2*C(IR)
      B(IR)=B(IR)+CPHI3*C(IR)
      C(IR)=C(IR)*CPHI1
*
*   step size is changed from smaller step size DELY3 to larger step
*   size DELY4
*
      DO 31 K=IR2,N
      PROD=AV(K)*DY4
      A(K)=-DYM4-PROD
      B(K)=AU(K)*DELX+2.0*DYM4+DT
      C(K)=-DYM4+PROD
```

```
*   size DELY2
*
      DO 90 K=IP2,IQ
      PROD=V(K)*DY2
      A(K)=-DYE2-PROD
      B(K)=U(K)*DELX+2.0*DYE2+DT
      C(K)=-DYE2+PROD
 90   CONTINUE
      A(IQ)=A(IQ)+BPHI2*C(IQ)
      B(IQ)=B(IQ)+BPHI3*C(IQ)
      C(IQ)=C(IQ)*BPHI1
*
*   step size is changed from smaller step size DELY2 to larger step
*   size DELY3
*
      DO 81 K=IQ2,IR
      PROD=V(K)*DY3
      A(K)=-DYE3-PROD
      B(K)=U(K)*DELX+2.0*DYE3+DT
      C(K)=-DYE3+PROD
 81   CONTINUE
      A(IR)=A(IR)+CPHI2*C(IR)
      B(IR)=B(IR)+CPHI3*C(IR)
      C(IR)=C(IR)*CPHI1
*
*   step size is changed from smaller step size DELY3 to larger step
*   size DELY4
*
      DO 91 K=IR2,N
      PROD=V(K)*DY4
      A(K)=-DYE4-PROD
      B(K)=U(K)*DELX+2.0*DYE4+DT
      C(K)=-DYE4+PROD
 91   CONTINUE
      DO 100 K=1,N
      D(K)=U(K)*PTHET(K,L)*DELX+THETI(K,L)*DT
100   CONTINUE
      D(1)=D(1)-A(1)*THEO
      CALL TRIDI (A,B,C,THET,D,N)
*
*   calculation of Theta's (temperatures) at various Y-locations
*   is complete
* elements of TRIDIAGONAL coefficient matrix and right hand side
* vector for species equation (cf. eqn. 3.19) are calculated
```

```
*
      DO 150 K=1,IP
      PROD=V(K)*DY1
      A(K)=-DYC1-PROD
      B(K)=U(K)*DELX+2.0*DYC1+DT
      C(K)=-DYC1+PROD
150   CONTINUE
      A(IP)=A(IP)+APHI2*C(IP)
      B(IP)=B(IP)+APHI3*C(IP)
      C(IP)=C(IP)*APHI1
*
*  step size is changed from smaller step size DELY1 to larger step
*  size DELY2
*
      DO 160 K=IP2,N
      PROD=V(K)*DY2
      A(K)=-DYC2-PROD
      B(K)=U(K)*DELX+2.0*DYC2+DT
      C(K)=-DYC2+PROD
160   CONTINUE
      A(IQ)=A(IQ)+BPHI2*C(IQ)
      B(IQ)=B(IQ)+BPHI3*C(IQ)
      C(IQ)=C(IQ)*BPHI1
*
*  step size is changed from smaller step size DELY2 to larger step
*  size DELY3
*
      DO 151 K=IQ2,IR
      PROD=V(K)*DY3
      A(K)=-DYC3-PROD
      B(K)=U(K)*DELX+2.0*DYC3+DT
      C(K)=-DYC3+PROD
151   CONTINUE
      A(IR)=A(IR)+CPHI2*C(IR)
      B(IR)=B(IR)+CPHI3*C(IR)
      C(IR)=C(IR)*CPHI1
*
*  step size is changed from smaller step size DELY3 to larger step
*  size DELY4
*
      DO 161 K=IR2,N
      PROD=V(K)*DY4
      A(K)=-DYC4-PROD
      B(K)=U(K)*DELX+2.0*DYC4+DT
```

```
      C(K)=-DYC4+PROD
161   CONTINUE
      DO 170 K=1,N
      D(K)=U(K)*PCON(K,L)*DELX+CONI(K,L)*DT
170   CONTINUE
      D(1)=D(1)-A(1)*CONO
      CALL TRIDI (A,B,C,CON,D,N)
*
*  calculation of C's (concentration at variousY-locations is complete
* whether solutions for U's,V's,Theta's,C's during iterative procedure
* are converged or not is checked. if yes we move to next X-location
* downstream. if not the values calculated in this iteration are the
* guess values for the next iteration. underrelaxation is employed
* refer to section 3.3 for details
*
      CALL MAX(AU,U,N,EPS)
      IF (EPS.LT.SMALL)GO TO 500
800   DO 120 K=1,N
      AU(K)=AU(K)+ALP*(U(K)-AU(K))
      AV(K)=AV(K)+ALP*(V(K)-AV(K))
      ATHET(K)=ATHET(K)+ALP*(THET(K)-ATHET(K))
      ACON(K)=ACON(K)+ALP*(CON(K)-ACON(K))
120   CONTINUE
      GO TO 400
500   CALL MAX (AV,V,N,EPS)
      IF(EPS.GT.SMALL)GO TO 800
      CALL MAX (ATHET,THET,N,EPS)
      IF (EPS.GT.SMALL)GO TO 800
      CALL MAX(ACON,CON,N,EPS)
      IF(EPS.GT.SMALL)GO TO 800
*
*  the instantaneous local Nusselt and Sherwood numbers are calculatec
*  solving the equations (3.13a) and (3.13b) respectively
*
      NUX(L+1)=(3.0-4.0*THET(1)+THET(2))*DY1
      SHX(L+1)=(3.0-4.0*CON(1)+CON(2))*DY1
*
*   writing out the values of U,V,Theta and C at every Y-location but
*   at X = 1.0 i. e. at the upper edge of the plate
*
      IF (J.NE.50) GO TO 786
      WRITE (25,97)X,TAU,ALP,ITR
97    FORMAT(2X,'X=',F3.1,1X,'TIME = ',F4.2,1X,'RELAX FACTOR = ',F3.1
      1,1X,'ITR = ',I2,/)
```

```
C(K)=-DYC4+PROD
CONTINUE
DO 170 K=1,N
D(K)=U(K)*PCON(K,L)*DELX+CONI(K,L)*DT
CONTINUE
D(1)=D(1)-A(1)*CONO
CALL TRIDI (A,B,C,CON,D,N)

lculation of C's (concentration at variousY-locations is complete
ther solutions for U's,V's,Theta's,C's during iterative procedure
 converged or not is checked. if yes we move to next X-location
nstream. if not the values calculated in this iteration are the
ss values for the next iteration. underrelaxation is employed
er to section 3.3 for details

CALL MAX(AU,U,N,EPS)
IF (EPS.LT.SMALL)GO TO 500
DO 120 K=1,N
AU(K)=AU(K)+ALP*(U(K)-AU(K))
AV(K)=AV(K)+ALP*(V(K)-AV(K))
ATHET(K)=ATHET(K)+ALP*(THET(K)-ATHET(K))
ACON(K)=ACON(K)+ALP*(CON(K)-ACON(K))
CONTINUE
GO TO 400
CALL MAX (AV,V,N,EPS)
IF(EPS.GT.SMALL)GO TO 800
CALL MAX (ATHET,THET,N,EPS)
IF (EPS.GT.SMALL)GO TO 800
CALL MAX(ACON,CON,N,EPS)
IF(EPS.GT.SMALL)GO TO 800

e instantaneous local Nusselt and Sherwood numbers are calculated
lving the equations (3.13a) and (3.13b) respectively

NUX(L+1)=(3.0-4.0*THET(1)+THET(2))*DY1
SHX(L+1)=(3.0-4.0*CON(1)+CON(2))*DY1

riting out the values of U,V,Theta and C at every Y-location but
t X = 1.0 i. e. at the upper edge of the plate

IF (J.NE.50) GO TO 786
WRITE (25,97)X,TAU,ALP,ITR
FORMAT(2X,'X=',F3.1,1X,'TIME = ',F4.2,1X,'RELAX FACTOR = ',F3.1
1,1X,'ITR = ',I2,/)
```

```
      WRITE(25,98)
      FORMAT(6X,'K',4X,'Y',5X,'U',6X,'V',7X,'THET',5X,'CON')
      WRITE(25,93)
      FORMAT(2X,45(1H-))
      WRITE(25,99)(K,Y(K),U(K),V(K),THET(K),CON(K),K=1,N)
      FORMAT(I7,F6.2,F8.4,F8.4,F8.4,F8.4)
      DO 130 K=1,N
      AU(K)=U(K)$AV(K)=V(K)$ATHET(K)=THET(K)$ACON(K)=CON(K)
      PU(K,L+1)=U(K)$PTHET(K,L+1)=THET(K)$PCON(K,L+1)=CON(K)
      CONTINUE
      CONTINUE

olution has been calculated out till the upper leading edge. now
e instantaneous mean Nusselt and Sherwood numbers are calculated
olving the eqns.(3.16a) and (3.16b) respectively

      NUM=-NUX(M+1)
      SHM=-SHX(M+1)
      DO 132 L=2,M,2
      NUM=NUM+4.0*NUX(L)+2.0*NUX(L+1)
      SHM=SHM+4.0*SHX(L)+2.0*SHX(L+1)
      CONTINUE
      D6=1./(3.*DELX)
      NUM=NUM*D6
      SHM=SHM*D6
      WRITE(25,133)NUM ,SHM
      FORMAT(/,2X,'MEAN NUSSELT NO= ',F7.5,2X,'MEAN SHERWOOD NO= '
      1,F7.5)
      WRITE(23,889)TAU,NUM,SHM
      FORMAT(3X,F4.2,2X,'|',7X,F6.4,6X,'|',6X,F6.4)
      WRITE(25,89)
      FORMAT(2X,52(1H#))

check whether steady state is reached or not. if yes stop the
lculations. if not go to next time step

      DO 990 L=1,M
      DO 991 K=1,N
      A(K)=UI(K,L)
      B(K)=PU(K,L)
      CONTINUE
      CALL MAX(A,B,N,EPS)
      IF(EPS.GT.SSMALL) GO TO 993
      CONTINUE
```

```
C     ***********************************************************
      SUBROUTINE TRIDI(A,B,C,X,R,N)
C SOLUTION OF N TRIDIAGONAL TYPE EQUATIONS
C     A * X(J-1) + B * X(J) + C * X(J+1) = R        WHERE
C A IS WRITTEN FOR A(J),..,R FOR R(J).X IS THE SOLN VECTOR.
C ALL THE DIMENSIONED VARIABLES HAVE DIMENSION N.  HOWEVER,
C A(1)&C(N) ARE NOT DEFINED IN A TRIDIAGONAL SET.  VECTORS A
C & R ARE DESTROYED.
      REAL A(N),B(N),C(N),X(N),R(N),BN
      A(N)=A(N)/B(N)
      R(N)=R(N)/B(N)
      DO 10 I=3,N
      K=N+3-I
      J=K-1
      BN=1./(B(J)-A(K)*C(J))
      A(J)=A(J)*BN
   10    R(J)=(R(J)-C(J)*R(K))*BN
      X(1)=(R(1)-C(1)*R(2))/(B(1)-A(2)*C(1))
      DO 20 I=2,N
   20    X(I)=R(I)-A(I)*X(I-1)
      RETURN
      END
C8**********************************************************
      SUBROUTINE MAX(X,Y,N,EPS)
      DIMENSION  X(N),Y(N)
      EPS=0.0
      DO 10 I=1,N
      EPSL=ABS(X(I)-Y(I))
      EPS=AMAX1(EPS,EPSL)
 10   CONTINUE
      RETURN
      END
```

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